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**Abstract**

Technical analysis (TA) is modeled as a method to infer market liquidity demand. Risk-averse market makers supply immediacy to an informed trader and uninformed technical traders, who conduct TA and trade strategically, and to liquidity traders, who trade randomly. Price change is positively related to both market liquidity demand and its change (order imbalance) when technical traders trade. Informed and technical traders' technical trading (i.e., trading based on TA) tends to offset the previous order imbalance (similar to *asynchronized trading* in Grossman and Miller, 1988), and generates negative return autocorrelations. As the number of technical traders increases, price impact declines and price informativeness improves, but successive return autocovariances and autocorrelations become more negative.

**JEL classification:** D4, D82, D84, G11, G12, G14

**Keywords:** Technical analysis; market liquidity demand; risk-averse market makers; imperfect competition.

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All the technician is really claiming is that price action should reflect shifts in supply and demand. ——— **John J. Murphy**

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# I. Introduction

Technical analysis (TA), the use of historical prices and perhaps other market statistics (e.g., volumes) to make investment decisions, is a pervasive activity in modern financial markets. TA has been widely accepted and adopted by practitioners, including large institutional investors, such as hedge funds, mutual funds, and proprietary trading desks, who are usually considered sophisticated and rational and whose trades incur significant market impact. The survey evidence in Menkhoff (2010) shows that the vast majority of fund managers rely on TA and consider TA to be more important than fundamental analysis at a forecasting horizon of weeks. Some investors even believe that these market statistics alone (without any fundamental information) provide indicators of future price movements. For example, Covel (2005) cites examples of large and successful hedge funds who advocate the use of TA exclusively without learning any fundamental information. Various studies also document the profitability of trading strategies based on TA.[1]

Academics, however, have long been skeptical about the usefulness of TA, perhaps because it seems to be inconsistent with the efficient market hypothesis, which states that the current price is a sufficient statistic for future price movements. As a result, much of the theoretical literature attempts to address this concern and explores the informational role of TA in addition to the current price to facilitate the learning of private information about the fundamental.[2]

[1]See, e.g., Brock, Lakonishok, and LeBaron (1992), Lo, Mamaysky, and Wang (2000), Shynkevich (2012), and Han, Yang, and Zhou (2013).

[2]Brown and Jennings (1989) and Grundy and McNichols (1989) examine noisy rational expectations equilibria with two rounds of trade, in which some fundamental is unknown to all traders and traders receive signals that are informative of the asset fundamental. As a result of the noise in the current price, historical prices, in addition to the current price, allow more accurate inferences of the private signals. In a modified model, Blume, Easley, and O'Hara (1994) show that volume provides information about the signal quality

Instead of exploring the informational role of TA, we model TA as a method to infer the market liquidity demand, which is the net position of buyers and sellers who search for liquidity and represents the demand and supply imbalance. Uninformed liquidity providers (e.g., market makers), who maintain a continuous presence in the markets, supply immediacy services to these investors and accommodate their trades.[3] When the liquidity providers are risk averse, the equilibrium price of an asset comprises two components: a fair value component (their estimate of the fundamental value) and a transitory pricing error component (the compensation for their immediacy services of bearing risk), as in Grossman and Stiglitz (1980), Grossman and Miller (1988), and Subrahmanyam (1991). TA allows investors, even uninformed, to infer these two components of the price and thus the market liquidity demand.

We analyze a Kyle-type model in which trading occurs in one risk-free bond and one risky stock in the first two periods and the stock's liquidation value is realized and becomes public information in Period 3. Similar to Kyle, there exist informed traders, who have monopolistic access to a private observation of the ex post liquidation value of the stock, and liquidity traders, who trade randomly for exogenous purposes. Our framework differs from Kyle by two perspectives. First, there exists a continuum of competitive and identical market makers with one unit mass, who are risk averse and have both inventory and adverse selection concerns. Second, and more importantly, there exists a number of uninformed technical traders, who make their investment decisions based on historical prices. The informed traders and technical traders compete on TA and trade strategically by taking account of the market impact of their trades. The informed, technical, and liquidity traders demand liquidity. As in Kyle, we call their aggregate trade "order flow". In our model, it is equivalent to "order imbalance" in Grossman and Miller (1988) and Chordia and Subrahmanyam (2004). These

that cannot be inferred from historical prices. Brunnermeier (2005) examines a Kyle-type model with two rounds of trade and risk-neutral market makers. An early-informed trader observes an imprecise signal about a forthcoming public announcement and exploits his signal twice, because his TA is more informative than that of the other traders.

[3] The typical textbooks on TA, e.g., Murphy (1999), emphasize the link between price movements and shifts in demand and supply for assets. The concept of liquidity providers is obviously suitable for an intermediated market. It, however, also makes sense if we interpret liquidity providers as those who passively accommodate trades initiated by other investors.

two terms are interchangable in our paper. Since we focus on the value of TA to uninformed traders, we normalize the number of informed traders to be one.

Compared with the literature, our model has three appealing features. First, it explains the usefulness of TA to uninformed traders without mixing with that of the current price, which is consistent with the original concept of TA. Second, it fits better to study large investors' competition on TA since both the informed trader and technical traders take into account the impact of their trades explicitly. Third, our model is tractable with risk-averse market makers. We are able to prove the existence of a linear equilibrium with imperfect competition on technical trading and obtain analytical solutions to study the behavior of heterogeneous investors rigorously.[4]

We show four main results. First, contrary to the Kyle-model, when technical traders trade, the informed trader trades against the deviation of the previous price from the liquidation value and on the previous price change; and price change (or return) is positively related to both the current-period order imbalance (which measures the change in market liquidity demand) and the accumulated order imbalance (which measures the market liquidity demand). Surprisingly, as the number of technical traders goes to infinity (let us say that there is free entry of technical traders), the coefficients on the order imbalance and the accumulated order imbalance converge to the temporary and permanent price impact, respectively.

The second result is that in Period 2, the informed and technical traders tend to offset the Period-1 order imbalance and are thus contrarians. In the case of free entry of technical traders, they completely offset the order imbalance. Grossman and Miller (1988) assume

[4]The term "imperfect competition" is adopted from Kyle (1989), as the informed and technical traders compete on TA and they recognize explicitly the impact of their trades on the equilibrium prices. It is usually hard to prove the existence of equilibrium in the literature on TA. For example, Brown and Jennings (1989), Grundy and McNichols (1989), and Blume, Easley, and O'Hara (1994) can prove the existence of equilibrium only in the special case in which myopic investors make their investment decisions on a period-to-period basis. In addition, prior to our work, it was widely believed that the tractability of equilibrium analysis is lost once risk-averse market markers are introduced into the Kyle-type dynamic model. For example, in Vayanos (2001), the equilibrium involves a large number of nonlinear equations and can only be solved with numerical calculations for general values of parameters.

asynchronization of the arrivals of buyers and sellers (usually called *asynchronized* trading) in a three-period model: New customers arrive in Period 2 to offset the order imbalance induced by a liquidity event in Period 1. They show that the risk of delayed trades creates the demand for immediacy. Our model, however, proves the inverse relation: As long as risk-averse liquidity providers supply immediacy (so that market prices are formed in each period), new customers infer the previous order imbalance using TA and tend to offset this imbalance, generating asynchronized trading.

The third result is that the technical trading of informed and technical traders induces a negative return autocorrelation between the first-two periods (i.e., price reversal). In contrast, the return autocorrelation is zero in the model with only liquidity traders and risk-averse market makers. The emphasis on large investors' exploitation of the transitory pricing errors using TA differentiates our model from the literature on price reversal, such as the bid-ask bounce (Roll, 1984, Jegadeesh and Titman, 1995), asynchronized trading (Grossman and Miller, 1988), mean-reverting noise supply (Campbell and Kyle, 1993), and market overreaction and correction (Cooper, 1999). This result is consistent with the recent empirical literature on the causal relationship between institutions' trading and return predictability, as in Edelen, Ince, and Kadlec (2016).

The forth result is that a more fierce competition on TA (proxied by an increase in the number of technical traders) has opposite effects on different measures of market illiquidity and efficiency.[5] Although price impact declines and price informativeness improves, the market becomes less liquid and less efficient in the sense that the return autocovariance and return autocorrelation between the first-two periods become more negative.

Our model also generates other interesting results. As the competition on TA intensifies, the market makers' intertemporal hedging demands get weaker and they trade more

[5]Price impact (the inverse of market depth) and negative return autocovariance are two widely used measures of illiquidity; price impact measures market impact per unit trade, while return autocovariance measures market impact of a whole trade (see, e.g., Vayanos and Wang, 2012). Price informativeness and return autocorrelation are two widely used measures of market efficiency; price informativeness measures the percentage of private information incorporated into price, while return autocorrelation measures the weak-form price efficiency (see, e.g., Campbell, Lo, and MacKinlay, 1997).

like short-term traders who make their investment decisions on a period-to-period basis; the informed trader can be either better or worse off, with his welfare measured by the unconditional expected profit. In the case of free entry of technical traders, the equilibrium simplifies to one in which the informed trader exclusively trades on his private information and market makers hold small inventories, which are determined by the current-period trades of the informed trader and liquidity traders.

Our model combines two lines of research on market makers in a dynamic setting: inventory models, such as Stoll (1978) and Grossman and Miller (1988), and asymmetric information models, such as Glosten and Milgrom (1985) and Kyle (1985). We show that when the market makers' risk aversion goes to zero, the informed trader trades exclusively on his private information, and our model converges to the Kyle model. On the other hand, when the market makers' risk aversion goes to infinity, the informed trader ignores his private information and trades exclusively on the transitory pricing error.

A few papers analyze strategic trading models of large investors with risk-averse market makers. Subrahmanyam (1991) develops a one-period model to investigate the effects of market makers' risk aversion on market liquidity and market efficiency. Vayanos (2001) analyzes the strategic trading of a large investor who has private information about his endowment shocks in the presence of noise traders and competitive risk-averse market makers in a stationary setting. Vayanos and Wang (2012) consider a model with two rounds of trade, in which agents are identical initially and become heterogenous and trade in the second period. They analyze how asymmetric information and competition affect market liquidity and asset prices. Guo and Kyle (2018) extend Guo and Ou-Yang (2015) by introducing risk-averse market makers and demonstrate with numerical calibrations that large informed traders' trades to exploit his informational advantages and the transitory pricing error generate both short-term momentum and long-term reversal. In contrast to these papers, our model suggests looking to TA and the competition on TA for a better understanding of market liquidity and asset price dynamics.

Zhu and Zhou (2009) also examine the usefulness of technical analysis to uninformed traders. They show that when there is uncertainty about the degree of predictability of the stock price, adding a moving averages (MA) component to the strategy that invests a fixed percentage of wealth in stock may increase investor utility. This is because MA is more robust to model and parameter misspecification. Their paper, however, does not relate the usefulness of TA to the detection of market liquidity demand. It is also unable to examine the effects of the competition on TA regarding market liquidity and asset price dynamics due to a partial equilibrium setting.

Two papers find empirical evidence on the link between TA and liquidity. Osler (2003) documents clustering in currency stop-loss and take-profit orders, and uses that clustering to explain two familiar predictions from TA: (1) trends tend to reverse course at predictable support and resistance levels, and (2) trends tend to be unusually rapid after exchange rates cross such levels. Kavajecz and Odders-White (2004) show the relation between TA and the change in the state of limit book movements. However, both papers do not develop theoretical models to explain the empirical evidence. They also do not examine how the competition on TA affects the equilibrium properties.

Chordia, Roll, and Subrahmanyam (2002) and Chordia and Subrahmanyam (2004) also examine the relations between order imbalances and returns. Chordia and Subrahmanyam (2004) develop a dynamic model in which risk-averse market makers accommodate autocorrelated order imbalances. They show that daily return is positively related to contemporaneous order imbalance but negatively related to lagged order imbalance. In contrast to these papers, our framework aims to examine how the competition of the informed and technical traders on TA affects these relations and we assume random liquidity trading.

The remainder of the paper is organized as follows. Section II details the dynamic strategic trading model outlined above. Section III solves for the equilibrium by studying market makers' inference problem and traders' optimization problems. We then determine the equilibria and discuss some key equilibrium properties for a baseline model with only market makers and liquidity traders, for our model, and for its two special cases. Section IV conducts the equilibrium analysis. Section V studies the effects of an increase in the number of technical traders (proxy for a more fierce competition on TA) on price quality, return autocorrelation, and the informed trader's welfare, and a limiting case of free entry of technical traders. Section VI discusses the empirical implications. Section VII concludes the paper. All proofs and figures are presented in the Appendix.

# II. The Model

In this section, we formally present a three-period model to analyze large investors' trading based on technical analysis (TA). In the economy, a risk-free bond and a risky stock are available for trading. For simplicity, we normalize the interest rate of the bond and the initial endowments of all traders in bond and stock to be zero. We model two trading periods, $t = 1$ and $t = 2$. The stock's liquidation value $D$ is realized and becomes public information in Period 3, and it is normally distributed with mean $p_0 = 0$ and variance $\sigma_D^2$. The Period-3 terminal price $p_3$ is thus given by $p_3 = D$. However, the intuition generalizes to many periods.

Four types of traders participate in the markets: informed traders, uninformed technical traders, liquidity traders, and market makers. Before trading, the informed traders observe the stock's liquidation value, but the rest only know its distribution. The technical traders observe historical prices and liquidity traders trade for exogenous motives. All traders liquidate their holdings and consume wealth in Period 3.

The informed traders make their investment decisions based on their private information and the historical prices, while the technical traders make their investment decisions only based on historical prices. Both trade strategically by taking into account the price impact cost. We can interpret them respectively as informed and uninformed large investors using TA in practice. To focus on the benefits of TA to uninformed traders, we normalize the number of informed traders to be one and set the number of technical traders to be $n$, where $n \geq 0$. For tractability, we assume that both types of traders are risk neutral and maximize their expected terminal wealth. Risk neutrality is appropriate since we are interested primarily in these traders' speculative trading using TA as opposed to, e.g., hedging consideration. Adopting this assumption also enables us to obtain closed-form solutions. Nevertheless, numerical analysis suggests that the main results are qualitatively the same when these traders are risk averse.

In each period $t \in \{1, 2\}$, the informed trader, each technical trader $i \in \{1, \cdots, n\}$, and liquidity traders submit respectively their market orders $x_t$, $z_{it}$, and $u_t$ to the market makers, where $u_1$ and $u_2$ are normally distributed with mean zero and variance $\sigma_u^2$, and are mutually independent and independent of the stock's liquidation value $D$. Random liquidity trading enables us to concentrate on the impact of the informed and technical traders' trades on the equilibrium prices. Since these traders initiate trading, they are liquidity demanders. Denote the order flow $\omega_t$ by $\omega_t = x_t + \sum_{i=1}^{n} z_{it} + u_t$ when $n \geq 1$ and $\omega_t = x_t + u_t$ when $n = 0$. Order imbalance is usually defined as the negative of the market makers' trade in an interval, as in Grossman and Miller (1988) and Chordia and Subrahmanyam (2004). In our model, order imbalance and order flow are the same. We do not distinguish between these two terms throughout the paper.

The market makers show a continuous presence in the market and supply immediacy to other traders. In each period, they set the stock price $p_t$ competitively based on the current and historical order flows and they accommodate the other traders' trades. To ensure that technical trading (trading based on TA) is profitable ex ante, we assume that the market makers are risk averse.[6] Similar to Vayanos (2001), there exists a continuum of

[6]Risk aversion is standard in inventory models and competitive trading models, and is a tractable way to model market makers' limited risk-bearing capacity. Adrian, Etula, and Shin (2015) show that when risk-neutral market makers provide immediacy services under the funding or value-at-risk (VaR) constraints, they are effectively risk averse. The empirical evidence in Chordia and Subrahmanyam (2004) and Hendershott and Seasholes (2007) suggests the existence of inventory effect. In Kyle (1985), market makers are risk neutral. Since they have exploited any profitable opportunities conditional on information from historical prices, investors do not profit from TA.

competitive and identical market makers with a unit mass. Each market maker is tiny and has a negative exponential utility function $-\exp(-\gamma_m W_3^m)$, where $\gamma_m$ is the common risk aversion coefficient and $W_3^m$ is his terminal wealth in Period 3. Market makers' aggregate demand is then integrated across each market maker's demand. An alternative interpretation is that a single competitive market maker absorbs the order imbalance, as in Subrahmanyam (1991).

Because the technical traders are identical and the informed and technical traders compete on TA by recognizing explicitly the impact of their trades on the equilibrium prices, we consider a symmetric Bayesian Nash equilibrium with imperfect competition on TA that satisfies the following two conditions.[7] First, given the market makers' updated beliefs about the stock's liquidation value and the trading strategies of other traders, the informed and each technical trader conduct TA and choose their optimal trading strategies strategically to maximize their expected profits. Second, in each period, given the informed and technical traders' trading strategies, market makers set the stock price competitively based on the order flows they observe in the current and previous periods. The market makers maximize their expected utilities and they accommodate the other traders' trades. Formally, it is given by a strategy profile $\{x_1^*, x_2^*, \{z_{i1}^*, z_{i2}^*\}_{\{i=1,...,n\}}, y_1^*, y_2^*, p_1^*(\cdot), p_2^*(\cdot)\}$, such that

$$
\begin{aligned}
1. \quad x_2^* &= \arg\max_{x_2} \quad E\left[x_1(D-p_1) + x_2(D-p_2)\,|D, \mathcal{F}_1, x_1\right] \\
x_1^* &= \arg\max_{x_1} \quad x_1(D - E[p_1|D]) + E\left[x_2^*(D-p_2)\,|D\right], \\
2. \quad z_{i2}^* &= \arg\max_{z_{i2}} \quad E\left[z_{i1}(D-p_1) + z_{i2}(D-p_2)\,|\mathcal{F}_1, z_{i1}\right] \\
z_{i1}^* &= \arg\max_{z_{i1}} \quad E\left[z_{i1}(D-p_1) + +z_{i2}^*(D-p_2)\right], \quad \forall i \in \{1, ..., n\}, \\
3. \quad y_2^* &= \arg\max_{y_2} \quad E\left[-\exp(-\gamma_m W_3^m)\,|\mathcal{F}_2, y_1\right] \\
y_1^* &= \arg\max_{y_1} \quad E\left[-\exp(-\gamma_m W_3^m)\,|\mathcal{F}_1, y_2 = y_2^*\right], \\
4. \quad p_1^* &= p_1(\omega_1^*) \text{ and } p_2^* = p_2(\omega_1^*, \omega_2^*),
\end{aligned}
$$

[7]We adopt the term "imperfect competition" from Kyle (1989).

where $\mathcal{F}_1 \equiv \{p_1\}$, $\mathcal{F}_2 \equiv \{p_1, p_2\}$, $W_3^m = y_1(D - p_1) + y_2(D - p_2)$, and the conditional expectations are derived using Bayesian rule such that the beliefs are consistent with the equilibrium strategies. Here, for simplicity, we consider only a representative market maker's optimization problem, where $\mathcal{F}_t$ denotes the market makers' information set in Period $t$. Given the pricing rule, $\mathcal{F}_1$ and $\mathcal{F}_2$ are equivalent to $\{\omega_1\}$ and $\{\omega_1, \omega_2\}$, respectively.

**Remark 1.** The literature has focused on how informed investors benefit from TA, as in Brown and Jennings (1989), Grundy and McNichols (1989), Blume, Easley, and O'Hara (1994), Hirshleifer, Subrahmanyam, and Titman (1994), Brunnermeier (2005), and Cespa and Vives (2012). In practice, however, uninformed investors also use TA to exploit trading opportunities. Introducing uninformed technical traders captures this feature.

**Remark 2.** Another possible framework to study large investors' competition on TA is Kyle (1989), in which all investors submit demand schedules. Adopting the framework in Kyle (1985), however, has two advantages. First, the model is analytically tractable; second, since the informed trader and uninformed technical traders make their investment decisions not contingent on the current-period price, their trading behavior is more consistent with the original concept of TA. See also the discussion in Blume, Easley, and O'Hara (1994) for the advantages of conditioning on historical rather than contemporaneous information.

# III. Equilibrium Determination

In this section, we determine an equilibrium. In Section III.A, we conjecture the stock prices and traders' equilibrium strategies. In Section III.B, we study market makers' inference problem. In Sections III.C, III.D, and III.E, we study traders' optimization problems. In Section III.F, we determine the equilibrium and discuss some key equilibrium properties for a baseline model with only market makers and liquidity traders, for our model, and for its two special cases.

## A. Stock Prices and Strategies

We first specify the equilibrium stock prices and the trading strategies of the informed and technical traders, respectively. We only consider a linear equilibrium.

We begin by postulating that the stock prices are given by

$$p_1 = \lambda_{11}\omega_1, \tag{1}$$

$$p_2 = \lambda_{21}\omega_1 + \lambda_{22}\omega_2, \tag{2}$$

where $\lambda_{11}$, $\lambda_{21}$, and $\lambda_{22}$ are positive constants, measuring price impact (per unit of trade).

We conjecture that the informed trader's trades are given by

$$x_1 = \beta_{11}D, \tag{3}$$

$$x_2 = \beta_{21}D + \beta_{22}\omega_1, \tag{4}$$

where $\beta_{11}$, $\beta_{21}$, and $\beta_{22}$ measure the informed trader's trading intensities (or aggressiveness).

Given his information in Period 2, each technical trader $i \in \{1, \cdots, n\}$ submits an order $z_{i2} = \alpha_{i2}\omega_1$ where $\alpha_{i2}$ measures his trading intensity. Because their orders are symmetric, their aggregate trade is

$$z_2 = \alpha_2\omega_1, \tag{5}$$

where $\alpha_2 = n\alpha_{i2}$. In Period 1, because the technical traders only know $p_0$, we assume that technical trader $i$'s and their aggregate trades are constants:

$$z_{i1} = \alpha_{i1}, \qquad z_1 = nz_{i1} = \alpha_1, \tag{6}$$

where $\alpha_1 = n\alpha_{i1}$.

## B. Market Makers' Inference

Given the information set $\mathcal{F}_t$, where $t \in \{1, 2\}$, the market makers' updated beliefs take the following forms:

$$E[D|\mathcal{F}_1] = \tau_{11}\omega_1, \tag{7}$$

$$E[D|\mathcal{F}_2] = \tau_{21}\omega_1 + \tau_{22}\omega_2, \tag{8}$$

where $\tau_{11}$, $\tau_{21}$, and $\tau_{22}$ capture the impacts of order flows on the market makers' estimates of the stock's liquidation value. Taking into account traders' trades (3), (4), (6), and (5) and applying the projection theorem for normally distributed random variables, we obtain

$$\tau_{11} = \frac{\beta_{11}\sigma_D^2}{c_0}, \tag{9}$$

$$\begin{pmatrix} \tau_{21} \\ \tau_{22} \end{pmatrix} = \begin{pmatrix} c_0 & c_1 \\ c_1 & c_2 \end{pmatrix}^{-1} \begin{pmatrix} \beta_{11}\sigma_D^2 \\ [\beta_{21} + \beta_{11}(\beta_{22} + \alpha_2)]\,\sigma_D^2 \end{pmatrix}. \tag{10}$$

where

$$c_0 = Var[\omega_1] = \beta_{11}^2\sigma_D^2 + \sigma_u^2, \tag{11}$$

$$c_1 = Cov[\omega_1, \omega_2] = E\big[\omega_1 E[\omega_2|\mathcal{F}_1]\big] = (\beta_{22} + \beta_{21}\tau_{11} + \alpha_2)c_0, \tag{12}$$

$$c_2 = E\big[Var[\omega_2|\mathcal{F}_1]\big] + Var\big[E[\omega_2|\mathcal{F}_1]\big] = (1 + \frac{\beta_{21}^2\sigma_d^2}{c_0})\sigma_u^2 + (\beta_{22} + \beta_{21}\tau_{11} + \alpha_2)^2 c_0. \tag{13}$$

Using the market makers' belief updating, the stock prices can be rewritten as:

$$p_1 = E[D|\mathcal{F}_1] + (\lambda_{11} - \tau_{11})\omega_1, \tag{14}$$

$$p_2 = E[D|\mathcal{F}_2] + (\lambda_{21} - \tau_{21})\omega_1 + (\lambda_{22} - \tau_{22})\omega_2. \tag{15}$$

The price in each trading period contains two components: a persistent and informational component, which is the market makers' estimate of the liquidation value conditional on their information sets and is usually termed as the stock's fair value ($E[D|\mathcal{F}_t]$, where $t \in \{1, 2\}$), and a transitory pricing error component, which compensates risk-averse market makers for holding undesired risky positions due to their limited risk-bearing capacities $[(\lambda_{11}-\tau_{11})\omega_1$ in Period 1 and $(\lambda_{21}-\tau_{21})\omega_1+(\lambda_{22}-\tau_{22})\omega_2$ in Period 2, respectively].[8] From (7) and (8), $\tau_{11}$, $\tau_{21}$, and $\tau_{22}$ measure price impact due to asymmetric information (*the permanent price impact)*, and $\lambda_{11}-\tau_{11}$, $\lambda_{21}-\tau_{21}$, and $\lambda_{22}-\tau_{22}$ measure price impact due to inventory risk (*the transitory price impact*).

Given the conjectured prices and competitive market makers' belief updating, we can then solve traders' optimization problems and obtain their trading strategies. In the ensuing analysis, we solve the linear equilibrium in which our conjectures are verified. The traders' optimization problems are derived using backward induction. Since there are no dividend payments in each period, the returns in Periods 1, 2, 3, are respectively given by $p_3-p_2=D-p_2$, $p_2-p_1$, and $p_1-p_0$, where $p_0$ is interpreted as the pre-trade price. However, the choice of the pre-trade price does not affect the results of our model in Sections III–V.

## C. *Market Makers' Maximization Problem*

Since market makers are identical and each of them is tiny, for simplicity, we solve for a representative market maker's optimization problem over her terminal wealth $W_3^m = y_1(D-p_1)+y_2(D-p_2)$. We can, of course, use a more complicated approach in which a market maker chooses her optimal trading strategy given that the other market makers adopt the equilibrium strategies. However, the results should be qualitatively the same.

In Period 2, the representative market maker chooses her position $y_2$ to maximize her expected terminal utility:

$$\begin{aligned} \max_{y_2} \quad & E\left[-\exp\left(-\gamma_m W_3^m\right)|\mathcal{F}_2\right], \\ s.t. \quad & W_3^m = y_1(D-p_1)+y_2(D-p_2). \end{aligned} \tag{16}$$

[8]We do not term this component as an illiquidity premium, because it can be either positive or negative, as opposed to the illiquidity premia in Amihud and Mendelson (1986) and others that are positive.

The FOC with respect to $y_2$ yields

$$y_2^* = \frac{E[D|\mathcal{F}_2] - p_2}{\gamma_m Var\left[D|\mathcal{F}_2\right]} - y_1. \tag{17}$$

In Period 1, the representative market maker chooses her position $y_1$ to maximize her expected terminal utility calculated at her optimal position in Period 2, $y_2^*$:

$$\begin{aligned} &\max_{y_1} \quad E\left[-\exp\left(-\gamma_m W_3^m\right)|\mathcal{F}_1\right], \\ &s.t. \quad W_3^m = y_1\left(p_2 - p_1\right) + \frac{\left(E[D|\mathcal{F}_2] - p_2\right)\left(D - p_2\right)}{\gamma_m Var\left[D|\mathcal{F}_2\right]}. \end{aligned} \tag{18}$$

The solution to this optimization problem is summarized in the following Lemma.

**LEMMA 1** *In Period 1, the representative market maker's optimal position is*

$$y_1^* = \frac{E\left[p_2|\mathcal{F}_1\right] - p_1}{\gamma_m Var\left[p_2|\mathcal{F}_1\right]} + \frac{IH}{\lambda_{11}} \times (p_1 - p_0), \tag{19}$$

*where*

$$IH \equiv -(\lambda_{22} - \tau_{22})\left(\lambda_{22} - \lambda_{21} + \lambda_{11}\right)/\lambda_{22}^2. \tag{20}$$

*Proof.* See Appendix A.

The market maker's Period-1 demand is more complicated and comprises two components. The first component $\frac{E[p_2|\mathcal{F}_1]-p_1}{\gamma_m Var[p_2|\mathcal{F}_1]}$ represents the demand of a short-term (or myopic) trader who makes her investment decision on a period by period basis (i.e., based on her estimate of the date-2 return), which is the focus of Brown and Jennings (1989), Grundy and McNichols (1989), and Blume, Easley, and O'Hara (1994). The second component $IH(p_1 - p_0)/\lambda_{11}$ represents her intertemporal hedging demand and $IH$ measures the sensitivity of her intertemporal hedging demand to order flow.

## D. *Informed Trader's Maximization Problem*

In Period 2, after observing Period-1 price $p_1$, the informed trader chooses $x_2$ to maximize his expected terminal wealth:

$$\max_{x_2} \quad E\left[x_1\left(D-p_1\right)+x_2\left(D-p_2\right)|D,\mathcal{F}_1\right],$$

Plugging in Period-2 price (2) and technical traders' aggregate order in Period 2 (5), the first-order condition (FOC) with respect to $x_2$ yields

$$x_2^* = \frac{D-\left(\lambda_{21}+\lambda_{22}\alpha_2\right)\omega_1}{2\lambda_{22}}. \tag{21}$$

Clearly, the second-order condition (SOC) is strictly negative when $\lambda_{22}>0$. Rearranging (21), we obtain $x_2^* = x_2^I + x_2^{NI}$, where

$$x_2^I \equiv \beta_{21}(D-E[D|\mathcal{F}_1]), \quad x_2^{NI} \;\equiv\; (\beta_{22}+\beta_{21}\tau_{11})\omega_1. \tag{22}$$

We term $x_2^I$ as the *informational component* and term $x_2^{NI}$ as the *non-informational component*.

In Period 1, using (21), the informed trader's maximization problem simplifies to

$$\max_{x_1} \quad x_1\left(D-E\left[p_1|D\right]\right)+\lambda_{22}E\left[\left(x_2^*\right)^2|D\right].$$

Substituting Period-1 price (1) and his Period-2 order (4), the FOC with respect to $x_1$ yields

$$x_1^* = \frac{1+2\lambda_{22}\beta_{21}\beta_{22}}{2\left(\lambda_{11}-\lambda_{22}\beta_{22}^2\right)}D. \tag{23}$$

As in Kyle (1985), to rule out the scheme in which the informed trader first destabilizes prices with an unprofitable trade and then makes a much more profitable trade in future periods, we require that the informed trader's SOC in Period 1 is negative, i.e.,

$$\lambda_{11} > \lambda_{22}\beta_{22}^2. \tag{24}$$

The informed trader's equilibrium trades (21) and (23) have the familiar expressions described by Kyle (1985), except that in our framework, he also takes into account the competition from the uninformed technical traders when exploiting the transitory pricing errors.

## E. *Technical Traders' Maximization Problem*

In Period 2, compared with the market makers, the technical traders observe the Period-1 price $p_1$ but not the Period-2 price $p_2$. Observing historical prices, however, allows them to infer the Period-1 order imbalance. Each technical trader $i \in \{1, \cdots, n\}$ submits $z_{i2}$ conditional on the information set $\mathcal{F}_1$ to maximize his expected terminal wealth:

$$\max_{z_{i2}} \quad E\left[z_{i1}\left(D - p_1\right) + z_{i2}\left(D - p_2\right)|\mathcal{F}_1\right].$$

Given the price (2), the informed trader's trade (4), the market makers' belief updating (7), and other technical traders' symmetric trades in Period 2, technical trader $i$ derives the FOC with respect to $z_{i2}$ and obtains

$$z_{i2}^* = \frac{\tau_{11} - \lambda_{21} - \lambda_{22}\left[\beta_{21}\tau_{11} + \beta_{22} + (n-1)\,\alpha_{i2}\right]}{2\lambda_{22}}\omega_1. \tag{25}$$

When $\lambda_{22} > 0$ is satisfied, as the requirement for the informed trader's maximization problem in Period 2, the SOC of each technical trader's problem is strictly negative.

In Period 1, the technical traders only know $p_0$. Using (25), the maximization problem of each technical trader $i \in \{1, \cdots, n\}$ is given by

$$\max_{z_{i1}} \quad E\left[z_{i1}\left(D - p_1\right) + \lambda_{22}\left(z_{i2}^*\right)^2\right].$$

Equivalently, given the price (1), the informed trader's trade (3), the market makers' belief updating (7), and other technical traders' symmetric trades in Period 1, each technical trader $i \in \{1, \cdots, n\}$ maximizes

$$\max_{z_{i1}} \quad \left(-\lambda_{11} + \alpha_i^2\lambda_{22}\right)\left(z_{i1}\right)^2. \tag{26}$$

To rule out the destablizing scheme similar to that of the informed trader, we require that the technical trader's SOC in Period 1 is negative, i.e.,

$$\lambda_{11} > \lambda_{22}\alpha_i^2. \tag{27}$$

As a result, $\alpha_{i1} = 0$ and $\alpha_1 = 0$. If (27) was not satisfied, then technical traders would trade infinitely aggressively in Period 1, which could not hold in equilibrium. In what follows, we demonstrate that (24) and (27) hold in equilibrium.

## F. *Equilibrium*

In each period, because market makers supply immediacy to other traders, we have

$$y_1 = -\omega_1, \qquad y_2 = -\omega_2. \tag{28}$$

Substituting (17) and (19) into (28) confirms the conjectured linear prices in (1) and (2). The updating of the market makers' beliefs, the traders' optimal trading strategies, and the market clearing conditions together determine an equilibrium.

For convenience, we define $\rho$ by

$$\rho \equiv \gamma_m \sigma_D \sigma_u, \tag{29}$$

which can be interpreted as a measure of the aggregate risk in the economy, whereby $\sigma_D$, $\sigma_u$, and $\gamma_m$ measure the cash flow volatility, the amount of liquidity trading, and the market makers' risk aversion coefficient, respectively.

To examine the impact of the informed and technical traders' trades on the equilibrium, we first study a baseline model in which these traders are not trading such that $\omega_1 = u_1$ and $\omega_2 = u_2$. Because information is symmetric, the price impact has only the transitory component. This model is solved using (17), (19), and (28). We summarize the results in theorem 1.

**THEOREM 1** *In the model in which the informed and technical traders are not trading*

*($\alpha_{i1} = \alpha_{i2} = \beta_{11} = \beta_{21} = \beta_{22} = 0$), the equilibrium is uniquely characterized by:*

$$\begin{aligned} \tau_{11} &= \tau_{21} = \tau_{22} = 0, \quad c_0 = c_2 = \sigma_u^2, \quad c_1 = 0, \\ \lambda_{11} &= \lambda_{21} = \lambda_{22} = \gamma_M \sigma_D^2 = \rho \frac{\sigma_D}{\sigma_u}. \end{aligned} \tag{30}$$

*Proof.* See Appendix B.

When the informed and technical traders are not trading, order flows have zero autocorrelation and price impact $\lambda_{11}$, $\lambda_{21}$, and $\lambda_{22}$ are the same, depending only on the cash flow volatility $\sigma_D$. The reason is because liquidity trades $u_1$ and $u_2$ are mutually independent and independent of $D$, the amount of liquidity trading $\sigma_u$ does not affect price impact. Thus,

$$p_2 - p_1 = \lambda_{22}\omega_2. \tag{31}$$

Due to random liquidity trading, the stock prices follow a martingale between the first two periods,

$$E\left[p_2 - p_1 | \mathcal{F}_1\right] = 0. \tag{32}$$

More generally, in Appendix B, we show that because of random liquidity trading, even if new market makers are allowed to enter the market in Period 2 (they may either observe both Period-1 and Period-2 prices or only observe Period-1 price), (31) and (32) still hold. Using (20), simple calculations yield

$$IH = -1 < 0. \tag{33}$$

Market makers' intertemporal hedging demands (in Period 1) are negatively related to Period-1 return $p_1 - p_0$. This hedging effect is a result that the transitory pricing error disappears in Period 3. In Period 1, the market makers perceive that the future Period-3 return $p_3 - p_2$ is negatively related to the current-period return $p_1 - p_0$, controlling for the Period-2 return $p_2 - p_1$.

However, when the informed and technical traders trade, market makers also face an adverse selection problem and the price impact contains both a permanent and a transitory component. It becomes much more complicated to solve the equilibrium. We show soon that (31) and (32) no longer hold. Conjecture that the endogenous parameters are given by

$$\begin{aligned}
&\alpha_{i1} = \tilde{\alpha}_{i1},\ \ \alpha_{i2} = \tilde{\alpha}_{i2},\ \ \beta_{11} = \tilde{\beta}_{11}\frac{\sigma_u}{\sigma_D},\ \ \beta_{21} = \tilde{\beta}_{21}\frac{\sigma_u}{\sigma_D},\ \ \beta_{22} = \tilde{\beta}_{22},\\
&\lambda_{11} = \tilde{\lambda}_{11}\frac{\sigma_D}{\sigma_u},\ \ \lambda_{21} = \tilde{\lambda}_{21}\frac{\sigma_D}{\sigma_u},\ \ \lambda_{22} = \tilde{\lambda}_{22}\frac{\sigma_D}{\sigma_u},\ \ \tau_{11} = \tilde{\tau}_{11}\frac{\sigma_D}{\sigma_u},\\
&\tau_{21} = \tilde{\tau}_{21}\frac{\sigma_D}{\sigma_u},\ \ \tau_{22} = \tilde{\tau}_{22}\frac{\sigma_D}{\sigma_u},\ \ c_0 = \tilde{c}_0\sigma_u^2,\ \ c_1 = \tilde{c}_1\sigma_u^2,\ \ c_2 = \tilde{c}_2\sigma_u^2,
\end{aligned} \tag{34}$$

where the variables with tilde are functions of exogenous parameters $\rho$ and $n$. These variables are constants when the market makers are risk neutral, as shown later in Corollary 2.

In the dynamic strategic trading models, when market makers are risk averse, it is usually hard to obtain analytical solutions (see, e.g., Vayanos, 2001). Fortuitously, our model has an analytically tractable equilibrium. We show that there always exists a linear equilibrium in which solving the equilibrium is equivalent to solving a nonlinear function of $\tilde{\tau}_{22}$ and all endogenous parameters with tilde are functions of $\tilde{\tau}_{22}$, $\rho$, and $n$. The results are presented in the following theorem.

**THEOREM 2** *There exists a linear symmetric equilibrium, in which the equilibrium profile* $\{\tilde{\alpha}_{i1}, \tilde{\alpha}_{i2}, \tilde{\beta}_{11}, \tilde{\beta}_{21}, \tilde{\beta}_{22}, \tilde{c}_0, \tilde{c}_1, \tilde{c}_2, \tilde{\tau}_{11}, \tilde{\tau}_{21}, \tilde{\tau}_{22}, \tilde{\lambda}_{11}, \tilde{\lambda}_{21}, \tilde{\lambda}_{22}\}$ *is determined by a* $\tilde{\tau}_{22} \in (\frac{1}{2\sqrt{2+\rho^2}}, \frac{1}{2\sqrt{1+\rho^2}})$. *Specifically, the parameters of the informed and the technical traders' trading strategies are given by*

$$\tilde{\alpha}_{i1} = 0,\quad \tilde{\alpha}_{i2} = \frac{\tilde{\tau}_{11} - \tilde{\lambda}_{11}}{(n+2)\tilde{\lambda}_{22} - n\tilde{\tau}_{22}} = \frac{-2\rho\tilde{\tau}_{22}}{1 + 2\rho(n+1)\tilde{\tau}_{22}} \in (-\frac{1}{n+1}, 0), \tag{35}$$

$$\tilde{\beta}_{11} = \frac{\sqrt{1 - 4(1+\rho^2)\tilde{\tau}_{22}^2}}{2\tilde{\tau}_{22}},\quad \tilde{\beta}_{21} = \frac{1 - 2\rho\tilde{\tau}_{22}}{2\tilde{\tau}_{22}},\quad \tilde{\beta}_{22} = -\frac{\tilde{\tau}_{11}(1 - 2\rho\tilde{\tau}_{22})}{2\tilde{\tau}_{22}} + \tilde{\alpha}_{i2} < 0, \tag{36}$$

*the market makers' belief updating parameters satisfy*

$$\tilde{c}_0 = \frac{1}{4\tilde{\tau}_{22}^2} - \rho^2,\ \ \tilde{c}_1 = (n+1)\tilde{\alpha}_{i2}\tilde{c}_0 < 0,\ \ \tilde{c}_2 = \frac{2}{1 + 2\rho\tilde{\tau}_{22}} + (n+1)^2\tilde{\alpha}_{i2}^2\tilde{c}_0, \tag{37}$$

$$\tilde{\tau}_{11} = \frac{\tilde{\beta}_{11}}{\tilde{c}_0},\ \ \tilde{\tau}_{21} = \frac{\tilde{\beta}_{11}}{\tilde{c}_0} - (n+1)\tilde{\alpha}_{i2}\tilde{\tau}_{22}, \tag{38}$$

*the price impact is given by*

$$\tilde{\lambda}_{11} = \frac{1+\tilde{\beta}_{22}}{2\tilde{\beta}_{11}} + \frac{\tilde{\beta}_{22}^2\tilde{\tau}_{22}}{1-2\rho\tilde{\tau}_{22}}, \quad \tilde{\lambda}_{21} = \tilde{\tau}_{21} + \frac{2\rho\tilde{\tau}_{22}^2}{1-2\rho\tilde{\tau}_{22}}, \quad \tilde{\lambda}_{22} = \frac{\tilde{\tau}_{22}}{1-2\rho\tilde{\tau}_{22}}, \tag{39}$$

*and $\tilde{\tau}_{22}$ solves the following nonlinear equation:*

$$\tilde{\lambda}_{11} - \tilde{\lambda}_{21} - n\tilde{\alpha}_{i2}\tilde{\tau}_{22} = 0. \tag{40}$$

*The second-order conditions of the informed and technical traders are satisfied, i.e., $\lambda_{11} > \lambda_{22}\beta_{22}^2$, $\lambda_{11} > \lambda_{22}\alpha_{i2}^2$, and $\lambda_{22} > 0$.*

*Proof.* See Appendix C.

The technical traders' aggregate trading intensity $\alpha_2 = n\alpha_{i2} \in (-\frac{n}{n+1}, 0]$, where $n \geq 0$. Substituting the expressions for $\beta_{21}$ and $\beta_{22}$ into (22) yields the informed trader's non-informational trade, $x_2^{NI} = \alpha_{i2}\omega_1$. Plugging in the expression for $\alpha_{i2}$ gives $z_{i2} = \alpha_{i2}\omega_1 = -\frac{(\lambda_{11}-\tau_{11})\omega_1}{(n+2)\lambda_{22}-n\tau_{22}}$. From (15), the Period-1 transitory pricing error is given by $(\lambda_{11} - \tau_{11})\,\omega_1$. The informed and technical traders infer the Period-1 order imbalance from Period-1 price. Since $(n+2)\lambda_{22} - n\tau_{22} > 0$, both the informed and technical traders trade against the Period-1 transitory pricing error and their trades in Period 2 tend to offset the previous order imbalance $\omega_1$.

From (40), $\lambda_{21} \geq \lambda_{11}$, and particularly, $\lambda_{21} > \lambda_{11}$ when $n > 0$. When technical traders trade, the price impact of the Period-1 order imbalance in Period 1 is smaller than its price impact in Period 2, contrary to Kyle (1985). Furthermore, for $n \geq 0$, we also have

$$\lambda_{21} - \tau_{21} = \lambda_{22} - \tau_{22} = \lambda_{11} - \tau_{11} + \tilde{\alpha}_{i2}\tilde{\tau}_{22} < \lambda_{11} - \tau_{11}, \quad \tau_{11} - \tau_{21} = (n+1)\tilde{\alpha}_{i2}\tilde{\tau}_{22} < 0. \tag{41}$$

The transitory price impact of the Period-1 order imbalance in Period 1 is larger than its transitory price impact in Period 2, because the informed and technical traders' trading to exploit the transitory error in Period 2 brings the stock price closer to its fair value; the permanent price impact of the Period-1 order imbalance in Period 1 is smaller than that in

Period 2, because these traders' trades generate negatively autocorrelated order imbalances.[9]

In Kyle (1985), price changes are driven only by the contemporaneous order imbalances; this result also holds in the model with risk-averse market makers and liquidity traders who trade randomly (see Theorem 1). In our model, using (38), simple calculations yield

$$p_2 - p_1 = \lambda_{22}\omega_2 + (\lambda_{21} - \lambda_{11})\omega_1 = (\lambda_{22} + n\alpha_{i2}\tau_{22})\omega_2 - n\alpha_{i2}\tau_{22}(\omega_1 + \omega_2). \qquad (42)$$

When $n > 0$, $-n\alpha_{i2} > 0$ and $\lambda_{22} + \alpha_2\tau_{22} > 0$. Therefore, this result holds in our model without technical traders; in the presence of technical traders, the Period-2 price change, however, is positively related to both current order imbalance $\omega_2$ (measuring the change in market liquidity demand) and the accumulated order imbalance $\omega_1 + \omega_2$ (measuring the market liquidity demand).[10] The market makers' expected Period-2 return is then given by

$$E\left[p_2 - p_1|\mathcal{F}_1\right] = \left[(n+1)\lambda_{22}\alpha_{i2} + (\lambda_{21} - \lambda_{11})\right]\omega_1 = \left[\alpha_{i2}\lambda_{22} + n\alpha_{i2}(\lambda_{22} - \tau_{22})\right]\omega_1, \qquad (43)$$

where $\alpha_{i2}\lambda_{22} + n\alpha_{i2}(\lambda_{22} - \tau_{22}) < 0$. The stock prices no longer follow a martingale.

The relationship between price changes and order imbalances can be understood from (43). The informed and technical traders' total technical trading, $(n+1)\alpha_{i2}\omega_1$, brings the stock price back to the fair value, which compensates market makers for their immediacy services of bearing undesired inventory risk. Without the technical traders, the informed trader balances his monopolistic advantages over his private information and the transitory pricing error. The compensation level is appropriate to market makers. Thus, $\lambda_{21} = \lambda_{11}$ and the price change depends only on the contemporaneous order imbalance. However, when the technical traders trade, because of the competition on TA, the informed and technical

[9]Permanent rice impacts are obtained from (9) and (10). As in a multivariate linear regression, if the independent variables are negatively correlated, the regression coefficients are larger than those in univariate regressions.

[10]Market liquidity demand is defined as the net position of buyers and sellers who search for liquidity. Liquidity traders trade randomly in our model. A key assumption for the price change to depend on the accumulated order imbalance (or lagged order imbalances) in addition to the order imbalance is the existence of technical traders. In Chordia and Subrahmanyam (2004), the price change is positively related to the contemporaneous order imbalance but is negatively related to the lagged order imbalance. Their results is driven by persistent liquidity trading.

traders' trading induces a larger price movement and thus a compensation level more than that required by competitive market makers. In equilibrium, $\lambda_{21} - \lambda_{11}$ has to be positive to bring down the compensation to market makers to a proper level, and thus the price change depends on both the order imbalance and the accumulated order imbalance.

Using (20), we have

$$IH = -2\rho\tilde{\tau}_{22}\left[1 + n\alpha_{i2}(1 - 2\rho\tilde{\tau}_{22})\right] \in (-1, 0), \tag{44}$$

and particularly, $IH = -(\tilde{\lambda}_{22} - \tilde{\tau}_{22})/\tilde{\lambda}_{22} = -2\rho\tilde{\tau}_{22}$ for $n = 0$. In the model with only market makers and liquidity traders, $Cov[p_2 - p_1, p_1 - p_0] = 0$ and $IH = -1$. Hence, market makers' intertemporal hedging demands get weaker and they trade more like short-term investors in the presence of the informed and technical traders. Because these traders' trading to exploit the transitory pricing error in Period 2 brings the stock price back to its fair value (market makers' estimate of the liquidation value), the market makers are less concerned about the future price reversal, leading to a weaker intertemporal hedging effect.

From our numerical analysis, we suspect but are unable to prove the uniqueness of the equilibrium. We next examine a special case in which we can prove a unique linear equilibrium. Letting $\rho \to \infty$ in Theorem 2 yields the following results.

**COROLLARY 1** *As $\rho \to \infty$, there exists a unique limiting equilibrium in which $\tilde{\tau}_{22}$ is given by*

$$\tilde{\tau}_{22} \to \frac{1}{2\rho} - \frac{1}{4\rho^3} + \frac{a}{\rho^5}, \tag{45}$$

*where $a = \frac{3}{16} - \left[\frac{(n+1)(n+2)}{2(n^2+4n+3)}\right]^2$. The other endogenous parameters satisfy*

$$\begin{aligned}
\tilde{\alpha}_{i2} &\to -\frac{1}{n+2}, \quad \tilde{\beta}_{11} \to \frac{\sqrt{3-16a}}{2\rho}, \quad \tilde{\beta}_{21} \to \frac{1}{2\rho}, \quad \tilde{\beta}_{22} \to -\frac{1}{n+2}, \\
\tilde{\tau}_{11} &\to \left[\sqrt{\frac{3}{4} - 4a} + \frac{n}{n+2}\right]\frac{1}{\rho}, \quad \tilde{\tau}_{21} \to \frac{\sqrt{3-16a}}{2\rho}, \quad \tilde{\lambda}_{11} \to \rho, \quad \tilde{\lambda}_{21} \to \rho, \\
\tilde{\lambda}_{22} &\to \rho, \quad \tilde{c}_0 \to 1, \quad \tilde{c}_1 \to -\frac{n+1}{n+2}, \quad \tilde{c}_2 \to 1 + \frac{(n+1)^2}{(n+2)^2}.
\end{aligned}$$

*Proof.* See Appendix D.

Interestingly, as $\rho$ goes to infinity, the equilibrium is similar to the one with only the market makers and liquidity traders: The permanent price impact goes to zero, while the transitory price impact is in the order of $\rho$, and thus converges to infinity. Surprisingly, the informed trader only trades on the transitory pricing error and completely ignores his private information. The reason is that the benefit from trading on his private information is not enough to cover the incurred price impact cost, which tends to be infinity, as $\rho$ goes to infinity. Consequently, the market makers cannot learn private information from order flows, leading to zero permanent price impact, i.e., $\tau_{11} \to 0$, $\tau_{21} \to 0$, and $\tau_{22} \to 0$.

Before proceeding, it is important to examine whether the Kyle-model is a special case of ours, since the market makers are competitive and maximize their expected utilities in our model, whereas the market makers do not maximize anything and just earn zero expected profits in the Kyle-model. We consider the special case in which $\gamma_M = 0$ such that $\rho = 0$. Letting $L = \frac{1}{2\sqrt{1-4\tilde{\tau}_{22}^2}} \geq 1/2$ and $\rho = 0$ in Theorem 2 yields the following results.

**COROLLARY 2** *The equilibrium profile* $\{\tilde{\alpha}_{i1}, \tilde{\alpha}_{i2}, \tilde{\beta}_{11}, \tilde{\beta}_{21}, \tilde{\beta}_{22}, \tilde{c}_0, \tilde{c}_1, \tilde{c}_2, \tilde{\tau}_{11}, \tilde{\tau}_{21}, \tilde{\tau}_{22}, \tilde{\lambda}_{11}, \tilde{\lambda}_{21}, \tilde{\lambda}_{22}\}$ *is continuous at* $\rho = 0$. *The endogenous parameters are uniquely characterized by*

$$\begin{aligned}
&\tilde{\alpha}_{i2} = \tilde{\alpha}_{i1} = 0, \quad \tilde{\beta}_{11} = \frac{2L-1}{4L-1}\frac{1}{\tilde{\lambda}_{11}}, \quad \tilde{\beta}_{21} = \frac{1}{2\tilde{\lambda}_{22}}, \\
&\tilde{\beta}_{22} = -\frac{1}{2L}, \quad \tilde{c}_0 = \frac{1}{4\tilde{\tau}_{22}^2}, \quad \tilde{c}_1 = 0, \quad \tilde{c}_2 = 2, \\
&\tilde{\lambda}_{11} = \tilde{\lambda}_{21} = \tilde{\tau}_{11} = \tilde{\tau}_{21} = \frac{\sqrt{2L\left(2L-1\right)}}{4L-1}, \quad \tilde{\lambda}_{22} = \tilde{\tau}_{22} = \sqrt{\frac{L}{2\left(4L-1\right)}},
\end{aligned}$$

*and*

$$L = \frac{\tilde{\lambda}_{22}}{\tilde{\lambda}_{21}} = \frac{1}{6}\left[1 + 2\sqrt{7}\cos\left(\frac{1}{3}\left(\pi - \arctan\sqrt{27}\right)\right)\right] \approx 0.901$$

*is the unique solution to a cubic equation*

$$8L^3 - 4L^2 - 4L + 1 = 0, \tag{46}$$

*where* $L > 1/2$.

*Proof.* See Appendix E.

It can be verified that the parameter values are the same as these calculated directly from the three-period Kyle-model. In other words, the Kyle model is indeed a special case of ours.

When the market makers are risk neutral, by calculation, the equilibrium is characterized by the following features: (1) the stock prices satisfy the martingale property; (2) successive returns have zero autocorrelation; (3) the technical traders choose not to trade; (4) the informed trader's trades cannot be forecasted by past returns; (5) market makers' positions cannot forecast future returns. Intuitively, risk-neutral market makers have already exploited any profitable opportunity contingent on historical prices, and consequently, there is no gain from strategies purely based on past prices. We demonstrate in the next section that these properties no longer hold when the market makers are risk averse.

# IV. Equilibrium Analysis

Introducing risk-averse market makers and uninformed technical traders in our model enables us to examine the equilibrium properties related to TA and returns, in contrast to the Kyle model. In this section, we first investigate the trading patterns of the technical and informed traders. Second, we assess the trading pattern of market makers. Third, we explore the model's predictions regarding return autocorrelations. Since our purpose is to examine the signs of these relationships, for simplicity, we use covariance expressions in this section. We leave the study of the competition on technical trading (proxied by an increase in the number of technical traders) to the next section. Note that all the variables refer to their equilibrium values in Section IV–V, and '$*$' is omitted for simplicity.

## A. *Technical and Informed Traders' Trades and Returns*

We start by examining the informed and technical traders' trading behavior. We first study technical trader $i$'s trade $z_{i2}$ and the aggregate trade of technical traders $z_2 = \alpha_2\omega_1$.

Proposition 1 summarizes the relationships between their trades and returns.

**PROPOSITION 1** *Each technical trader's trade and their aggregate trade in Period 2 have the following properties:*

$$\begin{aligned} Cov\left[z_{i2}, p_1 - p_0\right] &= \frac{1}{n} Cov\left[z_2, p_1 - p_0\right] = \alpha_{i2}\lambda_{11}c_0 < 0, \\ Cov\left[z_{i2}, p_2 - p_1\right] &= \frac{1}{n} Cov\left[z_2, p_2 - p_1\right] = \alpha_{i2}\left[\left(\lambda_{21} - \lambda_{11}\right)c_0 + \lambda_{22}c_1\right] > 0, \\ Cov\left[z_{i2}, D - p_2\right] &= \frac{1}{n} Cov\left[z_2, D - p_2\right] = \alpha_{i2}\left[\left(\tau_{11} - \lambda_{21}\right)c_0 - \lambda_{22}c_1\right] > 0. \end{aligned}$$

*Proof.* See Appendix F.

When the market makers are risk averse, the technical traders employ a contrarian strategy; $z_{i2}$ and $z_2$ have positive market impacts and positively forecast future returns. The reason is as follows. The stock price $p_1$ deviates from its fair value (market makers' estimate of the liquidation value) by an amount proportional to the order imbalance, $\omega_1$, to compensate risk-averse market makers for bearing undesired inventory risk. Observing the Period-1 price, the technical traders can infer the stock's fair value, the transitory pricing error, and thus the order imbalance. Knowing that the deviation of the stock price from the liquidation value is only temporary, they tend to offset the order imbalance and are thus contrarian ($Cov\left[z_{i2}, p_1 - p_0\right] < 0$). Because their trades incur positive price impact, they choose not trade too aggressively to push the price completely back to the pre-trade fair value, so their trades positively forecast future returns ($Cov\left[z_{i2}, D - p_2\right] > 0$).

We next assess the trading patterns of the informed trader. To compare with the technical traders' trading, we restrict our attention to the informed trader's Period-2 trade. Proposition 2 characterizes the properties of $x_2$.

**PROPOSITION 2** *The informed trader's Period-2 trade has the following properties:*

$$\begin{aligned} Cov\left[x_2, p_1 - p_0\right] &= \lambda_{11}\alpha_{i2}c_0 < 0, \\ Cov\left[x_2, p_2 - p_1\right] &= \alpha_{i2}\left[\left(\lambda_{21} - \lambda_{11}\right)c_0 + \lambda_{22}c_1\right] + \beta_{21}^2 Var[D|\mathcal{F}_1] > 0, \end{aligned}$$

$$Cov\,[x_2, D - p_2] \;=\; -\alpha_{i2}(\lambda_{21} - \tau_{21})(c_0 + c_1) + \beta_{21}\big[Var[D|\mathcal{F}_1] + Var[D|\mathcal{F}_2]\big] > 0.$$

*Proof.* See Appendix G.

When the market makers are risk averse, the informed trader employs a contrarian strategy in Period 2; his trade has a positive price impact and positively forecasts future returns. Employing TA causes both the informed and technical traders to be contrarian.

Grossman and Miller (1988) consider a three-period model and assume asynchronization of the arrivals of buyers and sellers (usually called *asynchronized* trading): New customers arrive in Period 2 to offset the order imbalance induced by a liquidity event in Period 1. They show that the risk of delayed trades creates the demand for immediacy. However, it is unclear why new customers choose to offset the initial order imbalance in their model.

Our model prove the inverse relation: As long as risk-averse liquidity providers maintain a continuous presence in the market to provide immediacy services (so that market prices are formed in each period), new customers will use TA to infer the previous market liquidity demand and their speculative trades tend to satisfy this demand, generating asynchronized trading. To our knowledge, this endogenous asynchronized trading is new to the literature.

Before proceeding, we compare the trading strategies of informed traders in the different models on TA. In Brown and Jennings (1989) and Grundy and McNichols (1989), and Blume, Easley, and O'Hara (1994), an informed trader's Period-2 satisfies

$$x_2 = \beta_e(E[D|S, p_1, p_2] - p_2) = \beta_e(E[D|S, p_1, p_2] - p_1) - \beta_e(p_2 - p_1),$$

where $\beta_e > 0$ and $S$ is his signal. An informed trader trades against the deviation of the historical price $p_1$ from his estimate of the stock's liquidation value and against the current-period return $p_2 - p_1$. In our model, the informed trader's trade can be rewritten as

$$x_2 = \beta_{21}(D - p_1) + \beta_0(p_1 - p_0),$$

where $\beta_{21} > 0$, $\beta_0 = 0$ for $n = 0$, and $\beta_0 = \rho\tilde{\tau}_{22} + \frac{\alpha_{i2}(1+\rho\tilde{\tau}_{22})}{\lambda_{11}} > 0$ for $n > 0$. When the technical traders are absent, trading on the private information $D - E[D|\mathcal{F}_1]$ and against the transitory pricing error $-(E[D|\mathcal{F}_1] - p_1)$ are equally profitable to the informed trader. Hence, he only trades against the deviation of the historical price from the stock's liquidation value $-(D-p_1)$, as in Kyle (1985). However, when the technical traders trade, because of the competition from the technical traders on TA, it is less profitable to trade on the transitory pricing error. The informed trader will also trade on the historical return $p_1 - p_0$, which is proportional to $-(E[D|\mathcal{F}_1] - p_1)$, to reduce his trading on the transitory pricing error.

## B. *Market Makers' Positions and Returns*

A large volume of empirical work has studied the relationships between market makers' positions and returns. Using our model, we study the trading behavior of market makers and these relationships. The results are presented below.

**PROPOSITION 3** *The market makers' trading has the following properties:*

$$Cov\left[y_1, p_1 - p_0\right] = -\lambda_{11} c_0 < 0, \qquad Cov\left[y_1, p_2 - p_1\right] = \left(\lambda_{11} - \lambda_{21}\right) c_0 - \lambda_{22} c_1 > 0,$$

$$Cov\left[y_1 + y_2, p_1 - p_0\right] = -\lambda_{11}\left(c_0 + c_1\right) < 0,$$

$$Cov\left[y_1 + y_2, p_2 - p_1\right] = -\left[\left(\lambda_{21} - \lambda_{11}\right)\left(c_0 + c_1\right) + \lambda_{22}\left(c_1 + c_2\right)\right] < 0,$$

$$Cov\left[y_1 + y_2, D - p_2\right] = \left(\lambda_{21} - \tau_{21}\right)\left(c_0 + 2c_1 + c_2\right) > 0,$$

$$Cov\left[y_1, y_2\right] = c_1 < 0, \qquad Cov\left[y_1, y_1 + y_2\right] = c_0 + c_1 > 0.$$

*Proof.* See Appendix H.

The market makers' positions are negatively related with past and contemporaneous returns, and are positively related with subsequent returns. In addition, the autocorrelations of their positions and trades are positive and negative, respectively. These results are not surprising and conform to the predictions of the inventory models on market makers, in which risk-averse market makers are compensated for holding undesired positions from providing immediacy to the market.

It is natural to ask whether risk aversion of market makers is sufficient to generate these results and what are the roles played by the informed and technical traders. To address these questions, we examine the model in which only liquidity traders and market makers trade and obtain the results using Theorem 1. As long as the market makers are risk averse, since the price and the liquidation value coincide in Period 3, the signs of the relationships related to market makers' positions in Period 2, $y_1 + y_2$, are still the same as those in Proposition 3. However, the two relationships about $y_1$ are different:

$$Cov\left[y_1, p_2 - p_1\right] = 0, \qquad Cov\left[y_1, y_2\right] = 0.$$

Simple calculations using Theorem 2 also yield that when the technical traders are absent ($n = 0$), $Cov[y_1, p_2 - p_1] = -\lambda_{22}c_1 > 0$ and $Cov[y_1, y_2] = c_1 < 0$. If the informed trader only exploited his private information, then successive order flows should have zero autocorrelation ($c_1 = 0$), as in Kyle (1985). Thus, the forecasting power of market makers' Period-1 positions for Period-2 return is caused by the technical and informed traders' technical trading, i.e, their trading to exploit the transitory pricing errors using TA.

## C. *Return Autocovariances*

A large body of literature explores the predictability power of historical prices, as in Brock, Lakonishok, and LeBaron (1992), Lo, Mamaysky, and Wang (2000), Shynkevich (2012), and Han, Yang, and Zhou (2013), which illustrates that even pure technical trading can be profitable. An empirical regularity about the time series properties of short-term stock returns is that stock prices tend to revert over horizons ranging from a week to a month, i.e., stock returns are negatively autocorrelated. See, e.g., Jegadeesh (1990) and Lehmann (1990). Negative autocovariance of returns (i.e., price reversal) has also been proposed as a measure of illiquidity. For example, Roll (1984) links it to the bid-ask spread; Vayanos and Wang

(2012) link it to the transitory component of price impact. Note that in our model, $\lambda_{11} - \tau_{11}$, $\lambda_{21} - \tau_{21}$, and $\lambda_{22} - \tau_{22}$ also measure the transitory price impact. However, these measures concern the impact per unit trade, while negative autocovariance concerns the impact of an entire trade. The results are summarized below.

**PROPOSITION 4** *When the market makers are risk averse, successive return autocovariances satisfy*

$$Cov\left[p_1 - p_0, p_2 - p_1\right] = \lambda_{11}\left[\left(\lambda_{21} - \lambda_{11}\right)c_0 + \lambda_{22}c_1\right] < 0, \tag{47}$$

$$Cov\left[p_2 - p_1, D - p_2\right] = \left(\tau_{21} - \lambda_{21}\right)\left[\left(\lambda_{21} - \lambda_{11}\right)\left(c_0 + c_1\right) + \lambda_{22}\left(c_1 + c_2\right)\right] < 0. \tag{48}$$

*Proof.* See Appendix I.

Since the stock prices follow a martingale when market makers are risk neutral, our model is in line with the inventory models developed by Stoll (1978), Grossman and Miller (1988), Campbell and Kyle (1993), and Campbell, Grossman, and Wang (1993). The interest of our model is, however, the impact of the trading of the informed and technical traders on return predictability. For this purpose, we use Theorem 1 to calculate return autocovariances for the model in which only market makers and liquidity traders trade. Simple calculations yield

$$Cov\left[p_1 - p_0, p_2 - p_1\right] = 0, \tag{49}$$

$$Cov\left[p_2 - p_1, D - p_2\right] = -\rho^2\sigma_D^2 < 0. \tag{50}$$

Equation (50) shows that the return autocovariance between the last two periods is negative. This result is a natural consequence of the risk aversion of the market makers, given that the price and the liquidation value coincide in Period 3. However, (49) shows that the prices follow a martingale between the first two periods though market makers are risk averse, because liquidity trades are random and independent across periods.

Does the informed trader's trading on his private information cause this negative autocovariance? To address this question, we consider the model in which the technical traders are absent ($n = 0$). Using Theorem 2, we obtain that $Cov[p_1 - p_0, p_2 - p_1] = \lambda_{11}\lambda_{22}c_1 < 0$. If the informed trader traded only on his private information, the successive order flows should have zero autocorrelation ($c_1 = 0$). Hence, our model shows a causal relationship between informed and technical traders' technical trading and the negative return autocovariance over the first two periods.

The emphasis on large investors' exploitation of the transitory pricing errors using TA differentiates our model from the current literature on price reversal, such as the bid-ask bounce (Roll, 1984, Jegadeesh and Titman, 1995), asynchronized trading (Grossman and Miller, 1988), mean-reverting noise supply (Campbell and Kyle, 1993), and market overreaction and correction (Cooper, 1999). Vayanos and Wang (2012) consider a model with two rounds of trade, in which agents are identical initially but become heterogenous and trade in the second period. In their model, the price reversal is caused by the liquidity shock in Period 2 and the coincidence of the stock price and the liquidation value in Period 3. Because there is no liquidity shock in Period 1, the Period-1 price is a constant and the return autocorrelation between the first two periods is zero.

# V. Competition on TA

Market data, such as prices and volumes, gets cheaper to use over time. Not surprisingly, quantitative trading conditional only on market data has become increasingly popular in the past decades. In this section, we examine how a more fierce competition on TA (proxied by an increase in the number of technical traders $n$) affects the equilibrium properties. We also examine a limiting case in which the technical traders can freely enter the market ($n \to \infty$).

Theorem 2 illustrates that the variables with tilde are determined only by $n$ and $\rho$. In what follows, we show that most of the properties we investigate in this section are also determined only by these two parameters. Without loss of generality, we normalize $\sigma_D = 1$ and $\sigma_u = 1$, and vary $\rho$ (equivalently $\gamma_m$) and $n$, where $\rho \in [0, 1]$ and $n \in [0, 50]$.

## A. *Effectiveness of TA and Price Quality*

We first examine the impact of the number of technical traders on the effectiveness of TA. Figure 1 plots the aggregate trading intensity of the technical traders $\alpha_2$, the correlation between technical traders' trade and Period-3 return $Corr\left[z_2, D - p_2\right]$, and the correlation between Period-1 return and Period-3 return $Corr\left[p_1 - p_0, D - p_2\right]$ as functions of $n$ for different values of $\rho$. Simple calculations show that these variables only depend on $n$ and $\rho$.

Figure *1* illustrates that as $n$ increases, $\alpha_2$ becomes more negative and $Corr\left[z_2, D - p_2\right]$ and $-Corr\left[p_1 - p_0, D - p_2\right]$ become less positive. As in a standard Cournot competition model, a more fierce competition causes the technical traders as a whole to exploit the transitory pricing errors more aggressively. Consequently, the forecasting powers of the technical traders' aggregate order and the Period-1 return for the Period-3 return, decline. This implies that a more fierce competition on TA reduces the effectiveness of TA.

We then examine the impact of technical trading on price quality (the quality of the price discovery process). Following the literature, price quality is measured from three dimensions: price informativeness, price variability, and market illiquidity. We measure price informativeness by $PI = 1 - Var\left[D|\mathcal{F}_2\right]/\sigma_D^2$. Because $Var\left[D|\mathcal{F}_2\right]$ reflects the amount of remaining private information about the stock's liquidation value after trading, $PI$ measures the percentage of private information impounded into prices by the end of Period 2. A larger $PI$ indicates a more informative price. By calculation,

$$PI = 1 - \frac{\tilde{\lambda}_{22} - \tilde{\tau}_{22}}{\rho}. \tag{51}$$

When the informed and technical traders trade, simple calculations using Theorem 2 yield $PI = 1 - (2\tilde{\tau}_{22}^2)/(1 - 2\rho\tilde{\tau}_{22})$, depending only on $\rho$ and $n$.

We measure stock price variability by $PV = Var\left[D - p_2\right]/\sigma_D^2$, the Period-3 return variance normalized by the variance of the stock's liquidation value. By calculation, we have

$$PV = (1 - PI) + \left(\tilde{\lambda}_{22} - \tilde{\tau}_{22}\right)^2 (\tilde{c}_0 + 2\tilde{c}_1 + \tilde{c}_2). \tag{52}$$

$PV$ contains two terms. The first term is positively related to the percentage of the remaining private information (or negatively related to $PI$), while the second is positively related to the variation of the transitory pricing error in Period 2

. When the informed and technical traders trade, using Theorem 2, we obtain that $PV = \frac{2\tilde{\tau}_{22}^2}{1-2\rho\tilde{\tau}_{22}} + \frac{4\rho^2\tilde{\tau}_{22}^2\tilde{\lambda}_{22}^2\tilde{c}_0}{[1+2\rho(1+n)\tilde{\tau}_{22}]^2}$, which depends only on $\rho$ and $n$. For simplicity, we choose price impact $\lambda_{11}$ and $\lambda_{22}$ to measure market illiquidity of the two periods, respectively. From Theorem 2, $\tilde{\lambda}_{11}$ and $\tilde{\lambda}_{22}$ depend only on $\rho$ and $n$.

Figure 2 plots $PI$, $PV$, $\lambda_{11}$, and $\lambda_{22}$ against $n$ for different values of $\rho$. This figure illustrates that as $n$ increases, the market becomes more liquid, and prices are more informative and less volatile. In other words, technical trading enhances price quality. The technical traders as a whole trade more aggressively to exploit the transitory pricing errors using TA. Consequently, the informed trader trades more aggressively on his private information due to the competition on TA. It is not surprising that the stock price deviates less from its fair value, is less volatile and more informative. Although the price impact due to information asymmetry rises, because the decline in the transitory price impact dominates, the market becomes more liquid.

## B. *Return Autocorrelation*

Successive return autocorrelation has been widely used to test the random walk hypothesis and more generally the weak-form market efficiency. We next study the effects of competition in using TA on return autocorrelations. Because the price and the liquidation value mechanically coincide in Period 3, we examine the return autocorrelation between the first two periods:

$$Corr\left[p_1 - p_0, p_2 - p_1\right] \equiv \frac{Cov\left[p_1 - p_0, p_2 - p_1\right]}{\sqrt{Var\left[p_2 - p_1\right] Var\left[p_1 - p_0\right]}}. \tag{53}$$

Simple calculations show that $Corr\left[p_1 - p_0, p_2 - p_1\right]$ only depends on $\rho$ and $n$.

Panel A of Figure 3 plots $Corr\left[p_1 - p_0, p_2 - p_1\right]$ as a function of $n$ for different values of $\rho$.

To understand the intuition behind this result, we also plot its two parts $Cov\left[p_1 - p_0, p_2 - p_1\right]$ and $\sqrt{Var\left[p_2 - p_1\right] Var\left[p_1 - p_0\right]}$ as functions of $n$ in Panels B and C, respectively. Panels F and G illustrate the results for two specific values of $\rho$. Surprisingly, as $n$ increases and thus the competition intensifies, although market quality improves, return autocorrelation $Corr\left[p_1 - p_0, p_2 - p_1\right]$ becomes more negative. This figures also shows that the reasons for this are that the average return variance $\sqrt{Var\left[p_2 - p_1\right] Var\left[p_1 - p_0\right]}$ declines and the return autocovariance $Cov\left[p_1 - p_0, p_2 - p_1\right]$ becomes more negative.

As shown in Section V. A., more technical trading increases price informativeness and reduces return volatilities. We only need to explain the more negative return autocovariance. Using Theorem 2, rearranging (47) yields

$$Cov\left[p_1 - p_0, p_2 - p_1\right] = \left[\lambda_{11}\lambda_{22}(\frac{1}{n} + 2\rho\tilde{\tau}_{22})\right] \times (\alpha_2 c_0)\,. \tag{54}$$

An increase in $n$ induces two opposite effects. On the one hand, Section V. A. shows that price impact (per unit of trade) decreases with the number of technical traders, leading to a decline in $\lambda_{11}\lambda_{22}(\frac{1}{n}+2\rho\tilde{\tau}_{22})$. On the other hand, the technical traders trade more aggressively to exploit the transitory pricing errors ($\alpha_2$ becomes more negative as in Figure 1) and the informed trader trades more aggressively on his private information, leading to a more volatile Period-1 order flow ($c_0$ increases in Panel D). Recall that return autocovariance measures the impact of an entire trade. Because the later effect dominates, the return autocovariance becomes more negative.[11]

These results have important empirical implications. The conventional wisdom suggests that a smaller magnitude of successive return autocorrelation represents a more efficient market [See the discussion in chapter 2 of Campbell, Lo, and MacKinlay (1997)]. Our paper, however, shows that a larger magnitude of return autocorrelation does not necessarily indicate a less efficient market, because more technical trading results in a higher price quantity,

[11] As $n$ increases, $Corr\left[p_2 - p_1, D - p_2\right]$ also becomes more negative. We find that $Cov\left[p_2 - p_1, D - p_2\right]$ becomes less negative. Hence, the more negative autocorrelation between the last two periods is caused by the decline in the average volatility $\sqrt{Var[D - p_2]Var[p_2 - p_1]}$. The details are available upon request.

i.e., a more informative and liquid price and a lower price volatility.

The technical traders' trades ultimately satisfy the informed and liquidity traders' liquidity demands, particularly when the number of technical traders is large (see Section V.D. for the details), while market makers provide immediacy services. When there are more market makers, these measures of market liquidity and efficiency all get improved: Not only price impact and price volatility decline and price informativeness enhances, but also the return autocovariance and autocorrelation become less negative (the results are available upon request).

As $n$ increases, because more technical traders exploit the forecasting power of historical prices, historical prices are less able to forecast future return. Hence, market makers' intertemporal hedging demands get weaker ($IH$ becomes less negative in Panel E) and they trade more like short-term traders.

## C. *Informed Trader's Welfare*

We then examine how the competition on TA affects the informed trader's welfare, which is measured by the unconditional expected profits $V \equiv E[x_1\,(D - p_1) + x_2\,(D - p_2)]$. By calculation, $V$ can be expressed as $(\sigma_D \sigma_u)\tilde{V}$, where $\tilde{V}$ only depends on $\rho$ and $n$.

Given that the informed trader exploits both his private information and the transitory pricing error, we decompose $V$ into two components, i.e., $V = V^I + V^{NI}$:

$$V^I = E\left[x_1(D - p_1) + x_2^I(D - p_2)\right], \quad V^{NI} = -E\left[x_2^{NI}(p_2 - E[D\,|\mathcal{F}_2])\right], \tag{55}$$

where $x_2^I$ and $x_2^{NI}$ are defined in (22). The first component $V^I$ is the informed trading component, representing the expected profit from trading on his private information; the second component $V^{NI}$ is the technical trading component, representing the expected profit from exploiting the transitory pricing errors.

Figure 4 plots $V$, $V^I$, $V^{NI}$ as functions of $n$ for different values of $\rho$ in Panels A, B, and C. Panels D and E illustrate the results for two specific values of $\rho$. When $\rho$ is small, the informed trader's unconditional expected profit $V$ increases monotonically with $n$. However, when $\rho$ is large, $V$ decreases first and then increases with $n$. The implication is that a more fierce competition on TA can cause the informed trader to be either better or worse off.

Two countervailing effects contribute to the patterns in Figure 4. On the one hand, due to a more fierce competition from the technical traders, the informed trader profits less from exploiting the transitory pricing error, leading to a decline in $V^{NI}$. On the other hand, price impact declines and the market becomes more liquid. The informed trader trades more aggressively on his private information because of a more favorable price, leading to an increase in $V^I$.

When $\rho$ is small or when $\rho$ is large and the number of technical traders $n$ is large, the benefit from exploiting transitory pricing errors is small. The second effect dominates. As a result, the informed trader benefits more from trading at a more favorable price, leading to a monotonic relation between $V$ and $n$. When $\rho$ is large, particularly when the number of technical traders $n$ is small, the informed trader's profit is mainly driven by exploiting the transitory pricing errors. A more fierce competition leads to a more significant drop in the expected profit from exploiting transitory pricing errors, and the first effect dominates. Thus, his unconditional expected profit $V$ initially drops with $n$.

In Figures 1–4, a change in $\rho$ is caused by a change in $\gamma_m$. However, from the expressions of relevant variables, it can be seen that even when the change in $\rho$ is caused by a change in $\sigma_D$, most of the results discussed above still hold. For example, as the number of technical traders increases, when $\sigma_D$ is small, $V$ increases monotonically. When $\sigma_D$ is large, $V$ decreases first and increases afterwards, exhibiting a U-shaped pattern.

## D. *Free Entry of Technical Traders*

In the real world, market data is cheap to access. Hence, a large number of technical traders enter the market at a low cost. In the limiting case $n \to \infty$, let us say there is free entry of technical traders. We present the limiting results in the following proposition. For simplicity, we replace "$\to$" by "$=$".

**PROPOSITION 5** *With free entry of technical traders, the equilibrium is characterized by*

$$\alpha_2 = -1, \quad Cov\left[z_2, D - p_2\right] = 0, \quad Cov\left[p_1 - p_0, D - p_2\right] = 0,$$
$$\tau_{11} = \tau_{21} - \tau_{22}, \quad \lambda_{11} - \tau_{11} = \lambda_{21} - \tau_{21} = \lambda_{22} - \tau_{22}, \quad IH = -4\rho^2 \tilde{\tau}_{22}^2 < 0,$$
$$Cov\left[p_1 - p_0, p_2 - p_1\right] = -\lambda_{11} c_0 \left(\lambda_{22} - \tau_{22}\right) < 0,$$
$$Cov\left[p_2 - p_1, D - p_2\right] = \frac{2\left(\tau_{21} - \lambda_{21}\right)\lambda_{22}\sigma_u^2}{1 + 2\rho\tilde{\tau}_{22}} < 0.$$

This proposition is a direct result of Theorem 2 and propositions 1–4 by letting $n \to \infty$, thus we omit its proof. Technical traders' Period-2 trades completely offset the order imbalance in Period 1, generating *asynchronized* trading, Period-1 return has no forecasting power for Period-3 return, and the informed trader only trades on his remaining private information.

By calculation, $\omega_1 + \omega_2 = \beta_{21}(D - E[D|\mathcal{F}_1]) + u_2$. With free entry of technical traders, the equilibrium in each period is much simplified: The informed trader only trades on his private information and market makers' inventory positions are determined only by the current-period trades of the informed and liquidity traders. Because their positions are independent across periods, the transitory price impact $\lambda_{11} - \tau_{11}$, $\lambda_{21} - \tau_{21}$, $\lambda_{22} - \tau_{22}$, are equal to each other. Substituting $\alpha_2 = -1$ into (42) yields

$$p_2 - p_1 = (\lambda_{22} - \tau_{22})\omega_2 + \tau_{22}(\omega_1 + \omega_2). \tag{56}$$

Interestingly, in the limiting case, the coefficients on the order imbalance and the accumulated order imbalance converge to the temporary and permanent price impact, respectively.

Proposition 5 shows that the forecasting powers of the technical traders' trades and Period-1 return for Period-3 return tend to disappear as if in a perfect competitive economy. Also, the competition on TA does not eliminate successive return autocorrelations. As the number of technical traders goes to infinity, both $Cov\left[p_1 - p_0, p_2 - p_1\right]$ and $Cov\left[p_2 - p_1, D - p_2\right]$ converge to certain negative values. Since successive returns are negatively autocorrelated, the market makers perceive that the Period-3 return $D - p_2$ is negatively related to Period-1 return $P_1 - p_0$ controlling for the Period-2 return $p_2 - p_1$, leading to a negative $IH$.

# VI. Empirical Implications

In this section, we explore the implications of our model for empirical studies. These implications concern the trades of informed and technical traders, the trades and positions of market makers, the two measures of illiquidity (price impact and negative return autocovariance), and return autocorrelation.

## A. *Informed and Technical Traders*

Various empirical evidence has documented that institutions make their investment decisions contingent on historical prices (i.e., employ TA). Thus, we can proxy the informed and technical traders in our model by institutional investors. Propositions 1 and 2 of our model show that both informed and uninformed traders adopt contrarian strategies and their trades forecast future returns. Proposition 3 implies that the order flow is negatively correlated with the previous return. These predictions are supported by the empirical evidence. Chordia, Roll, and Subrahmanyam (2002) report that aggregate daily order imbalance on the NYSE increases after market declines and vice versa, indicating that investors are contrarians on aggregate. Jylha, Rinne, and Suominen (2014) find that hedge funds typically adopt contrarian strategies in the stock market. Chordia, Goyal, and Jegadeesh (2016) show that institutional investors behave as contrarians at daily and weekly frequencies. The informed trader in our model can also be represented by corporate insiders. Our findings match the empirical evidence that insiders often purchase (sell) shares after periods of negative (positive) abnormal stock performance and that insiders' trades can predict future stock returns. See, e.g., Seyhun (1986), Rozeff and Zaman (1998), Lakonishok and Lee (2001), Piotroski and Roulstone (2005), and Jenter (2005).

Our model further predicts that as the number of uninformed technical traders increases, the informed trader trades more on his private information and less from exploiting the transitory pricing errors; his expected profit increases monotonically when the cash flow volatility is small, but decreases first and increases afterwards otherwise. Also, the informed trader mainly exploits the transitory pricing error when the cash flow volatility is high; the informed trader not only trades against the deviation of the previous price from his estimated fundamental value, but trades on the previous price change when technical traders trade. A simple way to determine whether an institution is an informed trader or a technical trader is that when regressing future stock returns on an institution's current-period trades and historical returns, if this institution's trades do not provide additional explanatory power, it is more likely to be classified as a technical trader.

## B. *Market Makers*

Our model shows that market makers' inventory positions are negatively related with past and contemporaneous returns, and are positively related with subsequent returns. In addition, the autocorrelations of their positions and trades are positive and negative, respectively. These predictions are consistent with the empirical findings. Hasbrouck and Sofianos (1993) and Madhavan and Smidt (1993) find that the inventory autocorrelations of the NYSE specialists are positive over short horizons. Hasbrouck (1988) and Madhavan and Sofianos (1998) find that the trades of the NYSE specialists are negatively autocorrelated over short horizons. Hendershott and Seasholes (2007) find that the NYSE specialists' positions are negatively correlated with past and contemporaneous returns and are positively correlated with subsequent returns over daily and weekly horizons.

Our model further predicts that as the number of technical traders increases, market makers' intertemporal hedging demands become weaker. To test this prediction, we can use (19) to estimate a short-term trader's position (setting $IH = 0$) and then regress the market makers' aggregate position against it. Equation (19) suggests that the coefficient increases with the number of technical traders.

### C. *Market Illiquidity and Market Efficiency*

Both price impact and negative return autocovariance have been widely used by the empirical studies as measures of illiquidity, with the former concerning the impact per unit trade while the latter concerns the impact of the entire trade. Return Autocorrelation has been widely used to test the random walk hypothesis and more generally the weak-form market efficiency.

Our model establishes a causal relationship between the trading of the informed and technical traders and return autocorrelation (in the spirit of Proposition 4). This result is consistent with the recent empirical literature on the causal relationship between institutions' trading and return predictability, as in Edelen, Ince, and Kadlec (2016). Our model further suggests that as the number of technical traders increases, the price impact decreases, while the successive return autocorrelations become more negative.

## VII. Conclusion

In this paper, we analyze technical analysis (TA) in a dynamic Kyle-type model with risk-averse market makers and uninformed technical traders. This model is tractable and is important for several reasons.

First, TA is modeled as a method to infer market liquidity demand and hence is useful even to uninformed investors. Technical trading of the informed and technical traders endogenously generates asynchronized trading described in Grossman and Miller (1988). When technical traders trade, the price change depends on both the current-period order imbalance (which measures the change in market liquidity demand) and the accumulated order imbalance (which measures the market liquidity demand); the informed trader trades against the deviation of the previous price from the liquidation value and trades on the previous price change. With a large number of uninformed technical traders, the market makers hold small inventory positions in the sense that they only offset the current-period net trade of informed and liquidity traders. Second, our model is useful to study the effects of the competition among informed and uninformed traders on TA. As the number of technical traders increases (or the competition on TA intensifies), even though the market quality enhances, successive return autocorrelations become more negative. Also, the informed trader can be either better or worse off. Third, we establish a causal relationship between the trading of large investors, who employ TA to exploit the transitory pricing errors, and successive return autocorrelations.

Our study suggests a few directions for further work. First, instead of adopting random liquidity trading, we can analyze the effects of TA in the presence of persistent liquidity trading.[12] This extension should deliver different but empirically testable predictions about, e.g., successive return autocorrelations and the relations between price changes and order imbalances. Second, endogenizing the number of technical traders by assuming that they each have to invest in a costly technology will be helpful to understand the effects of one trader lowering the cost of technology, perhaps due to technological progress. Third, using our framework, we can study the effects of the imperfect competition among informed traders on the role of the stock price in aggregating information in a dynamic setting. The informed traders exploit heterogeneous private information about the stock's liquidation value and homogeneous transitory pricing errors, In contrast, Kyle (1989) examines this issue in a static (one-period) model in which each trader submits a demand schedule. Exploration of these issues is left for future research.

[12] Chordia and Subrahmanyam (2004) model persistent liquidity trading, motivated by the empirical evidence that institutional investors fulfil an order over a number of days.

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# Appendix

**A Proof of Lemma 1.** Using (17) and market clearing condition $y_2 = -\omega_2$, we obtain

$$\lambda_{21} = \tau_{21} + \gamma_m Var\left[D|\mathcal{F}_2\right], \quad \lambda_{22} = \tau_{22} + \gamma_m Var\left[D|\mathcal{F}_2\right]. \tag{A1}$$

Note that from (18), conditional on the market makers' Period-2 information set $\mathcal{F}_2$, $-\exp\left[-\gamma_m W_3^m\right]$ follows a log-normal distribution:

$$\begin{aligned} E\left[-\gamma_m W_3^m | \mathcal{F}_2\right] &= -\gamma_m \left[y_1 \left(p_2 - p_1\right) + \frac{\left(E\left[D|\mathcal{F}_2\right] - p_2\right)^2}{\gamma_m Var\left[D|\mathcal{F}_2\right]}\right], \\ Var\left[-\gamma_m W_3^m | \mathcal{F}_2\right] &= \frac{\left(E\left[D|\mathcal{F}_2\right] - p_2\right)^2}{2Var\left[D|\mathcal{F}_2\right]}. \end{aligned}$$

By the law of iterated expectations and (18), we have

$$\begin{aligned} E\big[-\exp\left[-\gamma_m W_3^m\right] | \mathcal{F}_1\big] &= E\big[\, E\left[-\exp\left[-\gamma_m W_3^m\right] | \mathcal{F}_2\right] | \mathcal{F}_1\big] \\ &\propto E\left[-\exp\left(-\gamma_m \left[y_1 \left(p_2 - p_1\right) + \frac{\left(E\left[D|\mathcal{F}_2\right] - p_2\right)^2}{2\gamma_m Var\left[D|\mathcal{F}_2\right]}\right]\right) \middle| \mathcal{F}_1\right]. \end{aligned} \tag{A2}$$

Denote $r_2$ by $r_2 \equiv p_2 - p_1 = \left(\lambda_{21} - \lambda_{11}\right)\omega_1 + \lambda_{22}\omega_2$. From (8), we have

$$E\left[D|\mathcal{F}_2\right] - p_2 = \left(\tau_{21} - \lambda_{21}\right)\omega_1 + \left(\tau_{22} - \lambda_{22}\right)\omega_2 = a_0 r_2 + a_1 \omega_1,$$

where

$$a_0 = \frac{\tau_{22} - \lambda_{22}}{\lambda_{22}}, \quad a_1 = \tau_{21} - \lambda_{21} - \frac{\left(\tau_{22} - \lambda_{22}\right)\left(\lambda_{21} - \lambda_{11}\right)}{\lambda_{22}}. \tag{A3}$$

Then (A2) becomes

$$\begin{aligned} & E\left[-\exp\left(-\gamma_m \left[y_1 r_2 + \frac{\left(a_0 r_2 + a_1 \omega_1\right)^2}{2\gamma_m Var\left[D|\mathcal{F}_2\right]}\right]\right) \middle| \mathcal{F}_1\right] \\ &= -\frac{1}{\sqrt{2\pi Var\left[p_2|\mathcal{F}_1\right]}} \int_{-\infty}^{\infty} \exp\left[-\gamma_m y_1 r_2 - \frac{\left(a_0 r_2 + a_1 \omega_1\right)^2}{2Var\left[D|\mathcal{F}_2\right]} - \frac{\left(r_2 - E\left[r_2|\mathcal{F}_1\right]\right)^2}{2Var\left[p_2|\mathcal{F}_1\right]}\right] dr_2 \\ &= -\frac{1}{\sqrt{2\pi Var\left[p_2|\mathcal{F}_1\right]}} \int_{-\infty}^{\infty} \exp\left[-b_0 \left(r_2 + \frac{b_1}{2b_0}\right)^2 + \frac{b_1^2}{4b_0^2} - b_2\right] dr_2 \\ &= -\frac{1}{\sqrt{2b_0 Var\left[p_2|\mathcal{F}_1\right]}} \exp\left(\frac{b_1^2}{4b_0^2} - b_2\right), \end{aligned} \tag{A4}$$

where $b_0 = \frac{1}{2Var[p_2|\mathcal{F}_1]} + \frac{a_0^2}{2Var[D|\mathcal{F}_2]}$, $b_1 = \gamma_m y_1 + \frac{a_0 a_1 \omega_1}{Var[D|\mathcal{F}_2]} - \frac{E[r_2|\mathcal{F}_1]}{Var[p_2|\mathcal{F}_1]}$, $b_2 = \frac{a_1^2 \omega_1^2}{2Var[D|\mathcal{F}_2]} + \frac{(E[r_2|\mathcal{F}_1])^2}{2Var[p_2|\mathcal{F}_1]}$.

The FOC of (A4) with respect to $y_1$ yields $b_1 = 0$, i.e.,

$$y_1^* = \frac{E\left[r_2|\mathcal{F}_1\right]}{\gamma_m Var\left[p_2|\mathcal{F}_1\right]} - \frac{a_0 a_1 \omega_1}{\gamma_m Var\left[D|\mathcal{F}_2\right]},$$

which can be rewritten as (19) because of (1), (A1), and (A3). Q.E.D.

**B Proof of Theorem 1.** We now assume that the informed and technical traders are not trading and hence information is symmetric. Then, $\beta_{11} = \beta_{21} = \beta_{22} = \alpha_{i2} = 0$. To compare with our model in which uninformed technical traders enter the market in Period 2, we consider a more general case in which existing market makers have a unit mass in Period 1 and the mass of new market makers who enter the market in Period 2 is $m$, where $m > 0$. From (17) and (28), we obtain that $\lambda_{21} = \lambda_{22} = \frac{\rho}{(1+m)} \frac{\sigma_D}{\sigma_u}$. Using (19) and (28) yields

$$(1 - \frac{\lambda_{11}}{\lambda_{22}}) = -\frac{1}{(1+m)}(1 - \frac{\lambda_{11}}{\lambda_{22}})\rho^2. \tag{A5}$$

We show that $\lambda_{11} = \lambda_{21}$ by contradiction. If $\lambda_{11} < \lambda_{21}$, then the left-hand side of (A5) is positive while the right-hand side is negative; if $\lambda_{11} > \lambda_{21}$, then the left-hand side is negative while the right-hand side is positive. Both cases are impossible. Hence, $\lambda_{11} = \lambda_{21} = \lambda_{22}$ in equilibrium.

We next consider the case in which the new market makers cannot trade conditional on the current-period price, i.e., they can only conduct TA. Denote their aggregate demand and original market makers' aggregate demand as $x_t$ and $x_m$, respectively. The first-order conditions (FOCs) in Period 2 give

$$x_t = \frac{-\lambda_{21} u_1}{\gamma_m \sigma_D^2}, \qquad x_m = \frac{-\lambda_{21} u_1 - \lambda_{22} u_2}{\gamma_m \sigma_D^2}. \tag{A6}$$

Market clearing condition in Period 2 yields $m x_t + x_m = -(u_1 + u_2)$. Plugging in the expressions for $x_t$ and $x_m$ and equating the coefficients on both sides yields

$$\lambda_{22} = \rho \frac{\sigma_D}{\sigma_u}, \qquad \lambda_{21} = \frac{\lambda_{22}}{1+m}.$$

The original market makers' FOC in Period 1 is still given by (19). Imposing the market clearing condition gives

$$(\frac{1}{1+m} - \frac{\lambda_{11}}{\lambda_{21}}) = -(\frac{1}{1+m} - \frac{\lambda_{11}}{\lambda_{21}})\rho^2. \tag{A7}$$

Again, since $\rho$ is an arbitrary non-negative number and (A7) always holds, $\lambda_{11} = \lambda_{21}$. To summarize, even if new market makers are allowed to enter the market in Period 2 (they either observe both Period-1 and Period-2 prices or only observe Period-1 price), the price impact of Period-1 liquidity trade is the same across these two periods. Because liquidity trades $u_1$ and $u_2$ are mutually independent and independent of $D$, $E[p_2|\mathcal{F}_1] = p_1$. Q.E.D.

**C Proof of Theorem 2.** For the informed trader's maximization problem, equating the coefficients of (4) and (21), we obtain

$$\beta_{21} = \frac{1}{2\lambda_{22}}, \quad \beta_{22} = -\frac{\lambda_{21} + \alpha_2 \lambda_{22}}{2\lambda_{22}}. \tag{A8}$$

Equating the coefficients of (3) and (23) delivers

$$\beta_{11} = \frac{1 + 2\lambda_{22}\beta_{21}\beta_{22}}{2\left(\lambda_{11} - \lambda_{22}\beta_{22}^2\right)} = \frac{1 + \beta_{22}}{2\left(\lambda_{11} - \lambda_{22}\beta_{22}^2\right)}. \tag{A9}$$

For each technical trader's symmetric maximization problem, equating the coefficients of (5) and

(25) yields

$$\alpha_{i2} = \frac{\tau_{11} - \lambda_{21} - \lambda_{22}\left[\beta_{21}\tau_{11} + \beta_{22} + (n-1)\,\alpha_{i2}\right]}{2\lambda_{22}}.$$

Plugging in (A8) and rearrangement gives

$$\alpha_{i2} = \frac{\tau_{11} - \lambda_{21}}{(n+2)\,\lambda_{22}}. \tag{A10}$$

In Period 1, as shown in the main text that, assuming (27) holds, the optimal solution for (26) is $z_{i1} = 0$.

Next, rearranging price function (2), market clearing condition $y_1 = -\omega_1$, and (19) yields

$$\lambda_{11} - \lambda_{21} - \lambda_{22}\frac{c_1}{c_0} = \gamma_m Var\left[p_2|\mathcal{F}_1\right]\left[1 - \frac{(\lambda_{22} - \tau_{22})\,(\lambda_{22} - \lambda_{21} + \lambda_{11})}{\lambda_{22}^2}\right]. \tag{A11}$$

Applying the projection theorem, we obtain

$$Var\left[p_2|\mathcal{F}_1\right] = \lambda_{22}^2 Var\left[\omega_2|\omega_1\right] = \lambda_{22}^2\left(c_2 - \frac{c_1^2}{c_0}\right),$$
$$Var\left[D|\mathcal{F}_2\right] = \sigma_D^2 - \left(c_0\tau_{21}^2 + 2c_1\tau_{21}\tau_{22} + c_2\tau_{22}^2\right).$$

Substituting into (A1) and (A11), simplification gives

$$\lambda_{11} - \lambda_{21} = \frac{\lambda_{22}\left[c_1 + \tau_{22}\gamma_m\left(c_0c_2 - c_1^2\right)\right]}{c_0 + \gamma_m\left(c_0c_2 - c_1^2\right)(\lambda_{22} - \tau_{22})}. \tag{A12}$$

Rearranging the price function (1), market clearing condition $y_2 = -\omega_2$, and (17) yields

$$\lambda_{21} - \tau_{21} = \lambda_{22} - \tau_{22} = \gamma_m\left[\sigma_D^2 - \left(c_0\tau_{21}^2 + 2c_1\tau_{21}\tau_{22} + c_2\tau_{22}^2\right)\right]. \tag{A13}$$

Conjecture that the parameters are given in (34). We first show that the endogenous parameters can be written as functions of $\tilde{\tau}_{22}$ and $rp_2 \equiv \tilde{\lambda}_{22} - \tilde{\tau}_{22}$. Note that $rp_2 = \tilde{\lambda}_{21} - \tilde{\tau}_{21}$. From (9), (10), and (12), we obtain

$$\tilde{\tau}_{11} = \tilde{\tau}_{21} + (n+1)\alpha_{i2}\tilde{\tau}_{22}. \tag{A14}$$

Substituting into (A10) and simplification yields

$$\alpha_{i2} = -\frac{1}{n+2} \times \frac{rp_2}{rp_2 + \tilde{\tau}_{22}/(n+2)}. \tag{A15}$$

From (11), (12), (13), (A8), (A14) and (A15), we can express $\tilde{c}_0$, $\tilde{c}_1$, $\tilde{c}_2$, $\tilde{\tau}_{21}$, $\tilde{\lambda}_{21}$, $\tilde{\beta}_{22}$, and $\tilde{\alpha}_{i2}$ as functions of $\tilde{\beta}_{11}$, $\tilde{\tau}_{22}$, and $rp_2$ (in addition to $\rho$ and $n$). Using the second equation in (10), we obtain

$$\tilde{\beta}_{11} = \frac{\sqrt{-\tilde{\tau}_{22}(8rp_2\tilde{\tau}_{22}^2 + 4\tilde{\tau}_{22}rp_2^2 + 4\tilde{\tau}_{22}^3 - \tilde{\tau}_{22} - 2rp_2)}}{2\tilde{\tau}_{22}(\tilde{\tau}_{22} + rp_2)}.$$

Thus, $\tilde{c}_0$, $\tilde{c}_1$, $\tilde{c}_2$, $\tilde{\tau}_{21}$, $\tilde{\lambda}_{21}$, $\tilde{\beta}_{22}$, and $\tilde{\alpha}_{i2}$ are functions of $\tilde{\tau}_{22}$ and $rp_2$. Substituting into (A13) yields

$$rp_2 = \frac{2\rho\tilde{\tau}_{22}^2}{1 - 2\rho\tilde{\tau}_{22}}.$$

Plugging the expression for $rp_2$ in the relevant equations gives the expressions for $\tilde{\alpha}_{i2}, \tilde{\beta}_{11}, \tilde{\beta}_{21}, \tilde{\beta}_{22}$, $\tilde{c}_0, \tilde{c}_1, \tilde{c}_2, \tilde{\tau}_{11}, \tilde{\tau}_{21}, \tilde{\lambda}_{11}, \tilde{\lambda}_{21}, \tilde{\lambda}_{22}$ as functions of $\tilde{\tau}_{22}$ in Theorem 2.

By calculation, we obtain

$$\frac{\tilde{\lambda}_{22}\left[c_1 + \tau_{22}\gamma_m\left(c_0c_2 - c_1^2\right)\right]}{c_0 + \gamma_m\left(c_0c_2 - c_1^2\right)\left(\lambda_{22} - \tau_{22}\right)} = \tilde{\lambda}_{22}\left(1 - 4\rho^2\tilde{\tau}_{22}^2\right)\left[(n+1)\tilde{\alpha}_{i2} + \rho\tilde{\tau}_{22}(1 + \frac{\tilde{\beta}_{21}^2}{\tilde{c}_0})\right].$$

Substituting into (A12) and simplification yields

$$\tilde{\lambda}_{11} - \tilde{\lambda}_{21} = n\tilde{\lambda}_{22}\left(1 - 2\rho\tilde{\tau}_{22}\right)\tilde{\alpha}_{i2} = n\tilde{\tau}_{22}\tilde{\alpha}_{i2}. \tag{A16}$$

Thus, (40) holds. Substituting into (A10) yields $\tilde{\alpha}_{i2} = \frac{\tilde{\tau}_{11} - \tilde{\lambda}_{11}}{(n+2)\tilde{\lambda}_{22} - n\tilde{\tau}_{22}}$.

Define

$$F(\rho, n, \tilde{\tau}_{22}) \equiv \frac{\tilde{\lambda}_{11} - \tilde{\lambda}_{21}}{\tilde{\tau}_{22}} - n\tilde{\alpha}_{i2}. \tag{A17}$$

When $\tilde{\tau}_{22} = \frac{1}{2\sqrt{2+\rho^2}}$, simple calculations yield $\tilde{c}_0 = 2$, $\tilde{\beta}_{11} = 1$, $\tilde{\tau}_{11} = 1/2$, $\tilde{\beta}_{21} = \sqrt{2+\rho^2} - \rho$, $\tilde{\alpha}_{i2} = \frac{-\rho}{\sqrt{2+\rho^2}+(n+1)\rho}$, $\tilde{\tau}_{21} = \frac{1}{2} - (n+1)\tilde{\alpha}_{i2}\tilde{\tau}_{22}$, $\tilde{\lambda}_{22} = \frac{\sqrt{2+\rho^2}+\rho}{4}$, and $\tilde{\beta}_{22} = -\frac{\tilde{\beta}_{11}}{2} + \tilde{\alpha}_{i2} > -\frac{1+\rho}{\sqrt{2+\rho^2}+\rho}$. Then,

$$F(\rho, n, \frac{1}{2\sqrt{2+\rho^2}}) = \frac{\tilde{\beta}_{22}}{2} + \tilde{\beta}_{22}^2\tilde{\lambda}_{22} + \tilde{\alpha}_{i2}\tilde{\tau}_{22} - 2\rho\tilde{\tau}_{22}\tilde{\lambda}_{22}. \tag{A18}$$

We prove $F(\rho, n, \frac{1}{2\sqrt{2+\rho^2}}) < 0$ by examining two separate cases. First, if $\rho \geq 1$, we have

$$\tilde{\beta}_{22}^2 - 2\rho\tilde{\tau}_{22} < \left(\frac{1+\rho}{\sqrt{2+\rho^2}+\rho}\right)^2 - \frac{\rho}{\sqrt{2+\rho^2}} = \frac{\sqrt{2+\rho^2}(1+2\rho-\rho^2) - 2\rho(1+\rho^2)}{\left(\sqrt{2+\rho^2}+\rho\right)^2\sqrt{2+\rho^2}}.$$

It can be verified that $(2+\rho^2)(1+2\rho-\rho^2)^2 - 4\rho^2(1+\rho^2)^2 < 0$ and thus $\tilde{\beta}_{22}^2 < 2\rho\tilde{\tau}_{22}$. Substituting into (A18) and using $\tilde{\beta}_{22} < 0$ and $\tilde{\alpha}_{i2} < 0$ yields that $F(\rho, n, \frac{1}{2\sqrt{2+\rho^2}}) < 0$. Second, if $0 \leq \rho < 1$, we have $1 + 2\tilde{\beta}_{22}\tilde{\lambda}_{22} > \frac{1-\rho}{2} > 0$. Substituting into (A18) and using $\tilde{\alpha}_{i2} < 0$ and $-2\rho\tilde{\tau}_{22}\tilde{\lambda}_{22} < 0$ gives $F(\rho, n, \frac{1}{2\sqrt{2+\rho^2}}) < 0$.

When $\tilde{\tau}_{22} = \frac{1}{2\sqrt{1+\rho^2}}$, simple calculations yield $\tilde{\beta}_{11} = 0$, $\tilde{c}_0 = 1$, $\tilde{\tau}_{21} = -(n+1)\tilde{\alpha}_{i2}\tilde{\tau}_{22}$, $\tilde{\tau}_{11} = 0$, $\tilde{\beta}_{22} = \tilde{\alpha}_{i2}$, and $\tilde{\lambda}_{11} \to +\infty$. Then, $F(\rho, n, \frac{1}{2\sqrt{1+\rho^2}}) \to +\infty$. Therefore, there exists a $\tilde{\tau}_{22} \in (\frac{1}{2\sqrt{2+\rho^2}}, \frac{1}{2\sqrt{1+\rho^2}})$ which solves (40).

We last show that (24) and (27) are satisfied. Note that

$$\tilde{\beta}_{22} = -\frac{\tilde{\tau}_{11}}{2\tilde{\lambda}_{22}} + \tilde{\alpha}_{i2} = \frac{\sqrt{1-4(1+\rho^2)\tilde{\tau}_{22}^2}}{1+2\rho\tilde{\tau}_{22}} - \frac{2\rho\tilde{\tau}_{22}}{1+2\rho(n+1)\tilde{\tau}_{22}} \geq \frac{\sqrt{1-4(1+\rho^2)\tilde{\tau}_{22}^2} - 2\rho\tilde{\tau}_{22}}{1+2\rho\tilde{\tau}_{22}} > -1.$$

Since $\tilde{\tau}_{11} > 0$, $\tilde{\beta}_{22}^2 > \tilde{\alpha}_{i2}^2$. Using (A9), we obtain

$$\tilde{\lambda}_{22}\tilde{\alpha}_{i2}^2 < \tilde{\lambda}_{22}\tilde{\beta}_{22}^2 = \tilde{\lambda}_{11} - \frac{1+\tilde{\beta}_{22}}{2\tilde{\beta}_{11}} < \tilde{\lambda}_{11}.$$

Thus, the SOCs of the informed and technical traders, (24) and (27), hold. It is straightforward to see $\lambda_{22} > 0$. Q.E.D.

**D Proof of Corollary 1.** We now determine the limiting equilibrium as $\rho \to \infty$. Since $\tilde{\tau}_{22} \in (\frac{1}{2\sqrt{2+\rho^2}}, \frac{1}{2\sqrt{1+\rho^2}})$, $\tilde{\tau}_{22} \sim O(1/\rho)$. As in Theorem 1, we first conjecture that $\lambda_{22} \sim O(\rho)$. Rearranging $\tilde{\lambda}_{22} = \tilde{\tau}_{22}/(1-2\rho\tilde{\tau}_{22})$ shows that $1 - 2\rho\tilde{\tau}_{22} = \tilde{\tau}_{22}/\tilde{\lambda}_{22} \sim O(1/\rho^2)$. Denote $\tilde{\tau}_{22}/\tilde{\lambda}_{22}$ by $\tilde{\tau}_{22}/\tilde{\lambda}_{22} \equiv 2a_0/\rho^2$, where $a_0$ is a constant. Thus, $\tilde{\tau}_{22} \to \frac{1}{2\rho} - \frac{a_0}{\rho^3}$ and $\tilde{\lambda}_{22} \to \frac{\rho}{4a_0}$. Conjecture that

$$\tilde{\tau}_{22} \to \frac{1}{2\rho} - \frac{a_0}{\rho^3} + \frac{a_1}{\rho^4} + \frac{a}{\rho^5}, \tag{A19}$$

Substituting (A19) into (36) and (37) gives

$$\begin{aligned}
\tilde{\alpha}_{i2} &\to \frac{-1}{n+2}, \quad \tilde{\beta}_{11} \to \sqrt{a_0 - \frac{1}{4} - \frac{a_1}{\rho} - \frac{a-a_0}{\rho^2}}, \quad \tilde{\beta}_{21} \to \frac{2a_0}{\rho}, \\
\tilde{\beta}_{22} &\to -\tilde{\beta}_{21}\tilde{\tau}_{11} - \frac{1}{n+2}, \quad \tilde{\tau}_{11} \to \frac{\tilde{\beta}_{11}}{4a_0}, \quad \tilde{\tau}_{21} \to \frac{\tilde{\beta}_{11}}{4a_0} + \frac{n+1}{2\rho(n+2)}, \\
\tilde{c}_0 &\to 4a_0, \quad \tilde{c}_1 \to -\frac{4(n+1)a_0}{n+2}, \quad \tilde{c}_2 \to 1 + 4a_0 \times \left(\frac{n+1}{n+2}\right)^2.
\end{aligned}$$

Since $\tilde{\beta}_{11} \sim o(\rho)$, $\tilde{\beta}_{22} \to -\frac{1}{n+2}$. We next prove that $a_0 = 1/4$ and $a_1 = 0$ by contradiction. Suppose that $a_0 \neq 1/4$ or $a_0 \neq 0$. From (39), we obtain

$$\frac{\tilde{\lambda}_{11} - \tilde{\lambda}_{21}}{\tilde{\lambda}_{22}} \to \frac{1}{(n+2)^2} - 1.$$

However, from (40), we obtain

$$\frac{\tilde{\lambda}_{11} - \tilde{\lambda}_{21}}{\tilde{\lambda}_{22}} \to 0.$$

These two conditions are contradicted with each other. Therefore, we must have $a_0 = 1/4$, $a_1 = 0$, and $\tilde{\beta}_{11} \to \sqrt{3/4 - 4a}/\rho$. Substituting the expressions for $\tilde{\lambda}_{11}$ and $\tilde{\lambda}_{21}$ into (40) yields

$$a = \frac{3}{16} - \left[\frac{(n+1)(n+2)}{2(n^2+4n+3)}\right]^2.$$

Substituting $a_0$, $a_1$, and $a$ into the expressions for the endogenous parameters yields the results in Corollary 1. Similar calculations show that $\lambda_{22}$ cannot be in other orders of $\rho$. Q.E.D.

**E Proof of Corollary 2.** We consider a special case of the model in which the market makers are risk neutral ($\gamma_m = 0$), such that $\rho = 0$. The results can be obtained by setting $\rho = 0$ in Theorem 2. Note that $\tilde{\alpha}_{i1}, \tilde{\alpha}_{i2}, \tilde{\beta}_{11}, \tilde{\beta}_{21}, \tilde{\beta}_{22}, \tilde{c}_0, \tilde{c}_1, \tilde{c}_2, \tilde{\tau}_{11}, \tilde{\tau}_{21}, \tilde{\lambda}_{11}, \tilde{\lambda}_{21}$, and $\tilde{\lambda}_{22}$ are all continuous at $\rho = 0$ controlling for $\tilde{\tau}_{22}$. Letting $\rho = 0$ in (36) yields

$$\tilde{\alpha}_{i2} = 0, \quad \tilde{\beta}_{11} = \frac{\sqrt{1-4\tilde{\tau}_{22}^2}}{2\tilde{\tau}_{22}}, \quad \tilde{\beta}_{21} = \frac{1}{2\tilde{\tau}_{22}}, \quad \tilde{\beta}_{22} = -\frac{\tilde{\tau}_{11}}{2\tilde{\tau}_{22}}. \tag{A20}$$

Letting $\rho = 0$ in (37), we obtain

$$\tilde{c}_0 = \frac{1}{4\tilde{\tau}_{22}^2}, \quad \tilde{c}_1 = 0, \quad \tilde{c}_2 = 2. \tag{A21}$$

Letting $\rho = 0$ in (38) and rearrangement gives

$$\tilde{\tau}_{11} = 2\tilde{\tau}_{22}\sqrt{1 - 4\tilde{\tau}_{22}^2}, \quad \tilde{\tau}_{21} = \frac{\tilde{\beta}_{11}}{\tilde{c}_0} = \tilde{\tau}_{11}. \tag{A22}$$

Letting $\rho = 0$ in (39) and Rearrangement yields

$$\tilde{\lambda}_{11} = \frac{1 + \tilde{\beta}_{22}}{2\tilde{\beta}_{11}} + \frac{\tilde{\tau}_{11}^2}{4\tilde{\tau}_{22}}, \quad \tilde{\lambda}_{21} = \tilde{\tau}_{21}, \quad \tilde{\lambda}_{22} = \tilde{\tau}_{22}. \tag{A23}$$

Letting $\rho = 0$ in (A17) yields that $\tilde{\tau}_{22}$ solves

$$F(n, \rho, \tilde{\tau}_{22}) = \frac{\tilde{\lambda}_{11} - \tilde{\tau}_{11}}{\tilde{\tau}_{22}} = 0. \tag{A24}$$

Define $L \equiv \frac{1}{2\sqrt{1-4\tilde{\tau}_{22}^2}}$. It is easy to see that $L \geq 0.5$ and $L$ is an increasing function of $\tilde{\tau}_{22}$ for $\tilde{\tau}_{22} > 0$. Plugging the corresponding expressions in (A24) and simplification yields

$$8L^3 - 4L^2 - 4L + 1 = 0. \tag{A25}$$

This cubic equation has three roots: $L_1 \approx 0.9010$, $L_2 \approx 0.2225$, and $L_3 \approx -0.6235$. Since $L > 0.5$, the unique root is $L = L_1$, equivalently $\tilde{\tau}_{22} = \sqrt{\frac{L_1}{2(4L_1-1)}} \approx 0.4159$.

We then prove that $\tilde{\tau}_{22}(\rho, n)$ is continuous at $\rho = 0$. We prove by contradiction. Assuming that $\lim_{\rho\to 0} \tilde{\tau}_{22}(\rho, n) = \tau_0 \neq \sqrt{\frac{L_1}{2(4L_1-1)}}$. Define $L_0$ as $L_0 = \frac{1}{2\sqrt{1-4\tau_0^2}} \geq 1/2$. Then, $L_0 \neq L_1$ and $\lim_{\rho\to 0} F\big(\rho, n, \tilde{\tau}_{22}(\rho, n)\big) = \lim_{\rho\to 0} [\lim_{\rho\to 0} F(\rho, n, \tilde{\tau}_{22})] = 8L_0^3 - 4L_0^2 - 4L_0 + 1$, where $\tilde{\tau}_{22}$ is held constant when calculating the limiting value inside the square bracket. Then $L_0$ is a root of (A25). Since there exists only one unique $L > 1/2$ satisfying (A25), $L_0 = L_1$, which is contradicted with our assumption. Thus, $\tilde{\tau}_{22}(\rho, n)$ is continuous at $\rho = 0$. Substituting the expression for $\tilde{\tau}_{22}$ into (A20), (A23), (A21), and (A24) yields the expressions for the other endogenous parameters in Corollary 2. Since the other endogenous parameters are functions of $\tilde{\tau}_{22}$, they are also continuous at $\rho = 0$.

Solving the original Kyle (1985) with risk-neutral market makers and two rounds of trade yields the same results as above. Therefore, the Kyle-model is a special case of our model. Q.E.D.

**F Proof of Proposition 1.** First, simple calculations yield the expressions in Proposition 1. From Theorem 2, $\alpha_2 < 0$, leading to $Cov[z_2, p_1 - p_0] < 0$. From (A16), we obtain

$$(\lambda_{21} - \lambda_{11})\, c_0 + \lambda_{22} c_1 = c_0 \lambda_{22} \alpha_2 (2\rho\tilde{\tau}_{22} + \frac{1}{n}) < 0. \tag{A26}$$

Thus, $Cov\,[z_2, p_2 - p_1] > 0$. Using (9) and (10), $\tau_{11} = \tau_{21} + c_1\tau_{22}/c_0$. Then,

$$(\tau_{11} - \lambda_{21})c_0 - \lambda_{22}c_1 = (\tau_{21} - \lambda_{21})c_0 - (\tau_{22} - \lambda_{22})c_1 < 0. \tag{A27}$$

Thus, $Cov\,[z_2, D - p_2] > 0$. Q.E.D.

**G Proof of Proposition 2.** Because $\alpha_{i2} < 0$, $Cov\,[x_2, p_1 - p_0] < 0$. Using Theorem 2 and $\tilde{\tau}_{11} \le \tilde{\tau}_{22} < \frac{1}{2\sqrt{1+\rho^2}}$, we obtain

$$\begin{aligned} Cov\,[x_2, p_2 - p_1] &= Cov\left[x_2^I + x_2^{NI}, (\lambda_{21} - \lambda_{11})\omega_1 + \lambda_{22}\omega_2\right] \\ &= \alpha_{i2}\left[(\lambda_{21} - \lambda_{11})\,c_0 + \lambda_{22}c_1\right] + \beta_{21}^2 Var[D|\mathcal{F}_1] > 0. \end{aligned}$$

From (A15), $-1/(n+1) \le \alpha_{i2} \le 0$. Thus,

$$c_0 + c_1 = [1 + (n+1)\alpha_{i2}]\,c_0 > 0. \tag{A28}$$

Simple calculations yield

$$Cov\,[x_2, D - p_2] = -\alpha_{i2}(\lambda_{21} - \tau_{21})(c_0 + c_1) + \beta_{21}\big[Var[D - E[D|\mathcal{F}_1]] + Var[D - E[D|\mathcal{F}_2]]\big] > 0.$$

The results in Proposition 2 are thus proved. Q.E.D.

**H Proof of Proposition 3.** Using (A27), we obtain $Cov\,[y_1, D - p_2] > 0$. Plugging in (A26) yields $Cov\,[y_1, p_2 - p_1] > 0$. From (A27), we have $Cov[y_1, D - p_2] > 0$. From (A28), $Cov\,[y_1, y_1 + y_2] > 0$ and $Cov\,[y_1 + y_2, p_1 - p_0] > 0$. Since $Var[\omega_1 + \omega_2] = c_0 + 2c_1 + c_2 > 0$, $Cov\,[y_1 + y_2, D - p_2] > 0$. Using Theorem 2, we obtain

$$\begin{aligned} \tilde{c}_1 + \tilde{c}_2 &= \left[(n+1)\alpha_{i2} + (n+1)^2\alpha_{i2}^2\right]\tilde{c}_0 + \frac{2}{1 + 2\rho\tilde{\tau}_{22}}, \\ &= \frac{2}{1 + 2\rho\tilde{\tau}_{22}} - \frac{2\rho(n+1)\tilde{\tau}_{22}\tilde{c}_0}{(1 + 2\rho(n+1)\tilde{\tau}_{22})} \times \frac{1}{(1 + 2\rho(n+1)\tilde{\tau}_{22})}. \end{aligned} \tag{A29}$$

Because $\tilde{\tau}_{22} > \frac{1}{2\sqrt{2+\rho^2}}$, $\tilde{c}_0 < 2$. Submitting this inequality into (A30) yields

$$\tilde{c}_1 + \tilde{c}_2 > \frac{2}{1 + 2\rho\tilde{\tau}_{22}} - \frac{2}{1 + 2\rho(n+1)\tilde{\tau}_{22}} > 0. \tag{A30}$$

From (A16), $\tilde{\lambda}_{11} - \tilde{\lambda}_{21} < 0$. Using (A28), we obtain that

$$Cov\,[y_1 + y_2, p_2 - p_1] = -\left[(\lambda_{21} - \lambda_{11})\,(c_0 + c_1) + \lambda_{22}\,(c_1 + c_2)\right] < 0. \tag{A31}$$

The results in Proposition 3 are thus proved. Q.E.D.

**I Proof of Proposition 4.** Using (A26) and (A31) respectively, we obtain

$$\begin{aligned} &Cov\,[p_1 - p_0, p_2 - p_1] = \lambda_{11}\left[(\lambda_{21} - \lambda_{11})\,c_0 + \lambda_{22}c_1\right] < 0, \\ &Cov\,[p_2 - p_1, D - p_2] = (\tau_{21} - \lambda_{21})\left[(\lambda_{21} - \lambda_{11})\,(c_0 + c_1) + \lambda_{22}\,(c_1 + c_2)\right] < 0. \end{aligned}$$

From (A27), we have $Cov\,[p_1 - p_0, D - p_2] < 0$. Q.E.D.

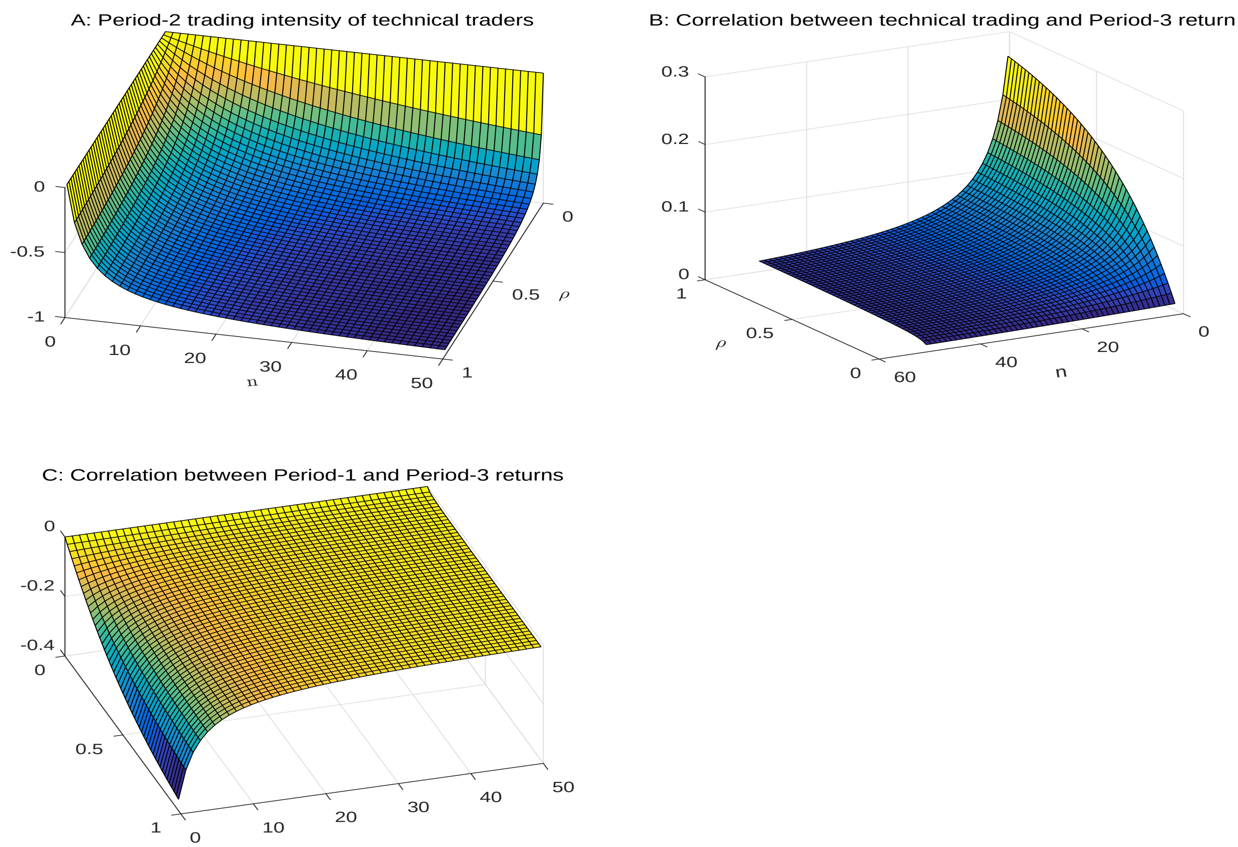


Figure 1: **Competition on TA and return predictability**. Panels A–C plot the technical traders' Period-2 aggregate trading intensity $\alpha_2$, the correlation between their trade and Period-3 return $Corr[z, D - p_2]$, and the correlation between Period-1 and Period-3 returns $Corr[p_1 - p_0, D - p_2]$ against the number of technical traders $n$ and the aggregate risk $\rho$, respectively. The parameter values for the figure are $\sigma_D = 1$ and $\sigma_u = 1$.

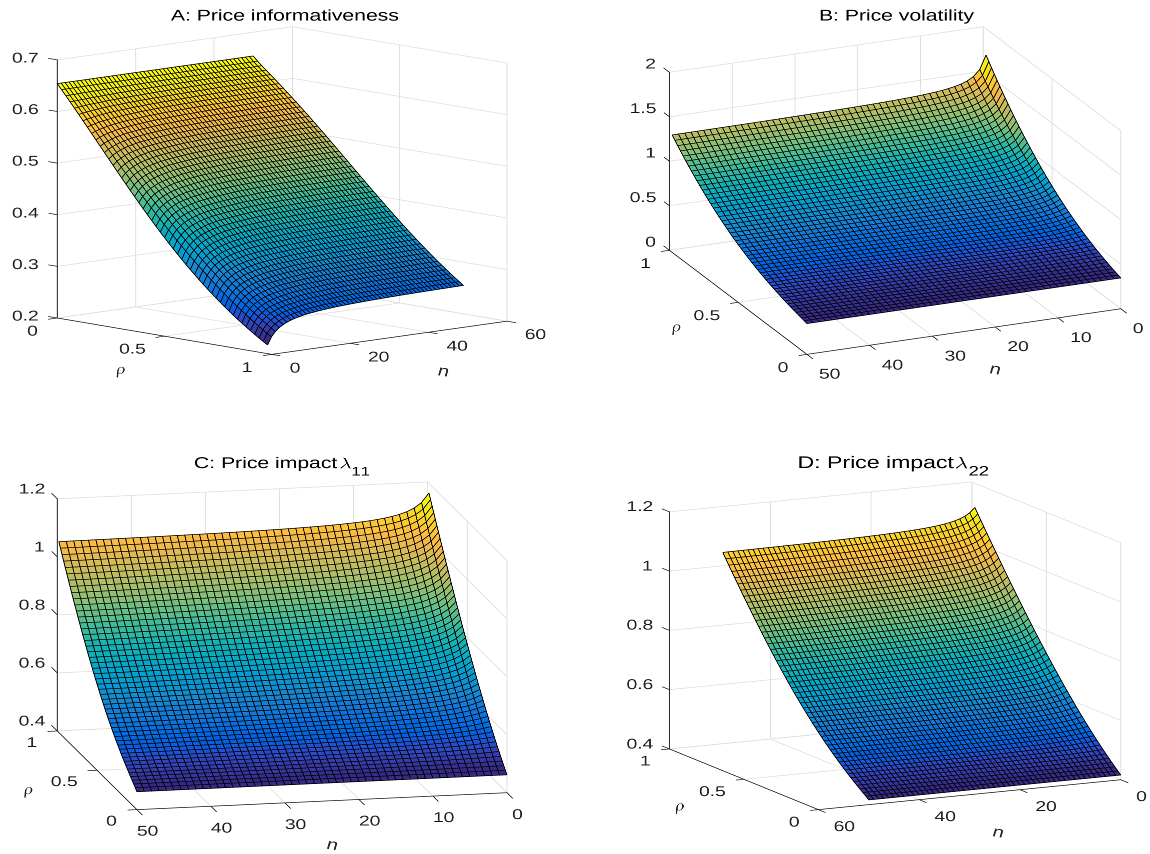


Figure 2: **Competition on TA and price quality**. Panels A–D plot price informativeness $PI$, price volatility $PV$, and illiquidity measures $\lambda_{11}$ and $\lambda_{22}$ against the number of technical traders $n$ and the aggregate risk $\rho$, respectively. The parameter values for the figure are $\sigma_D = 1$ and $\sigma_u = 1$.

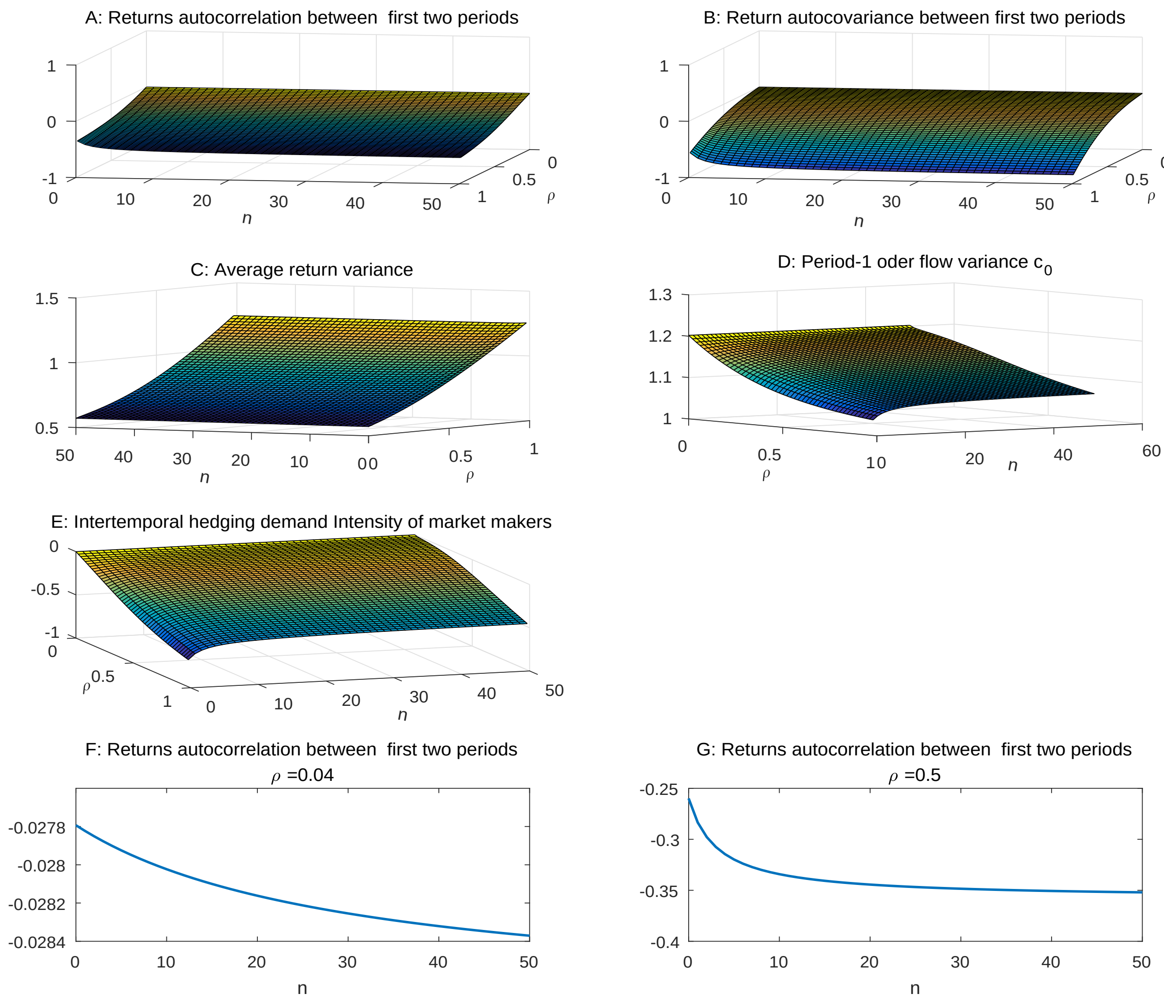


Figure 3: **Competition on TA and return autocorrelation**. Panels A–E plot the return autocorrelation between the first two periods $Corr[p_1 - p_0, p_2 - p_1]$, the return auto-covariance between the first two periods $Cov[p_1 - p_0, p_2 - p_1]$, the average return variance $\sqrt{Var\left[p_2 - p_1\right] Var\left[p_1 - p_0\right]}$, the Period-1 order flow variance $c_0$, and the sensitivity of market makers' intertemporal hedging demand to order flow $IH \equiv -\left(\lambda_{22} - \tau_{22}\right)\left(\lambda_{22} - \lambda_{21} + \lambda_{11}\right)/\lambda_{22}^2$ against the number of technical traders $n$ and the aggregate risk $\rho$. Panels F and G plot $Corr[p_1 - p_0, p_2 - p_1]$ against the number of technical traders $n$ for the aggregate risk $\rho = 0.04$ and $\rho = 0.5$, respectively. The parameter values for the figure are $\sigma_D = 1$ and $\sigma_u = 1$.

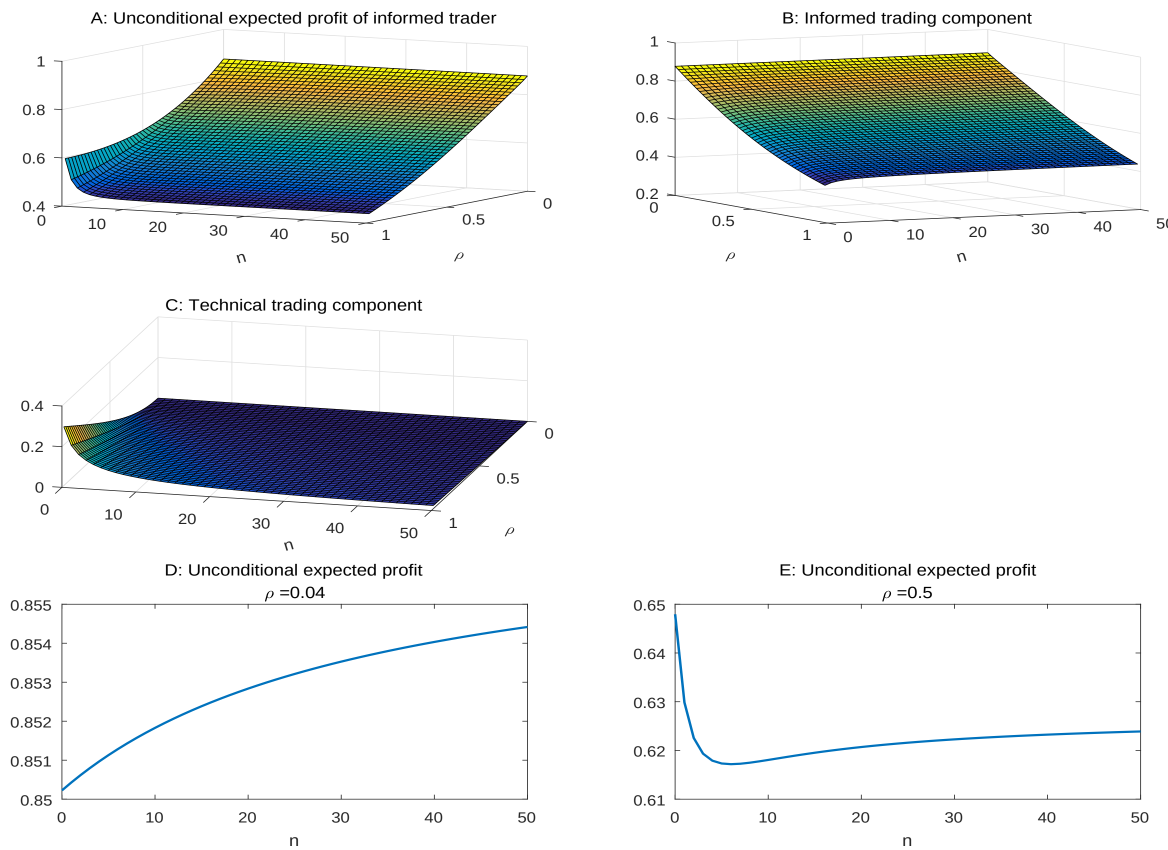


Figure 4: **Competition on TA and informed trader's welfare**. Panels A–C plot the informed trader's unconditional expected profit $E[x_1\,(D-p_1)+x_2\,(D-p_2)]$ and its two components defined in (55) against the number of technical traders $n$ and the aggregate risk $\rho$. Panels D and E plot the informed trader's unconditional expected profit against the number of technical traders $n$ for the aggregate risk $\rho=0.04$ and $\rho=0.5$, respectively. The parameter values for the figure are $\sigma_D=1$ and $\sigma_u=1$.