Net Neutrality and Welfare Effects of Discrimination on Either or Both the Content Side and the User Side

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Net Neutrality and Welfare Effects of Discrimination on Either or Both the Content Side and the User Side*

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Abstract

We study how net neutrality regulation on the content-provider side interacts with net neutrality on the internet-user side. We set up a theoretical model in which deviations from net neutrality take the form of price discrimination triggered either directly by asymmetries across content providers or indirectly by the content preferences of internet users. We show that unconstrained discrimination yields higher welfare than full net neutrality in which discrimination is banned on both sides of the market. However, net neutrality on the user side may dominate unconstrained discrimination. These results are driven by consumption reallocation effects within the content and user sides and by matching effects between them whose direction and relative intensity depend on the balance of the network effects.

Keywords: Net Neutrality, Zero-rating, Price Discrimination, Platform, Network Effects
JEL Classification Numbers: D4, L22, L43

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1 Introduction

Platforms that connect two different sides of a market can engage in price discrimination against either one side or both sides. Given interactions between the two sides, reflected by network effects, discrimination against one side should affect the other side. This paper examines how the different sides of a market interact when one or both are subject to discrimination, a process that ultimately affects consumer and social welfare.

Specifically, we focus on net neutrality, which is a high-profile case of non-discriminatory regulation in a two-sided market. Fundamentally, all deviations from net neutrality involve some form of discrimination regarding content. Specific to platforms in a two-sided market, content can be treated differentially by direct discrimination against content providers (CPs) and/or by indirect discrimination against internet users. In the former case, discrimination is based directly on the content’s characteristics, such as higher bandwidth consumption (cost-based motivation) or vertical affiliation with the ISP (own-content bias). This type of discrimination is enacted through paid prioritization, blocking, throttling, and congestion management on the content side. In contrast, users are subject to indirect discrimination based on the characteristics of the content they favor. For example, data consumption related to certain types of content may be zero-rated, i.e., not counted toward the user’s monthly data cap. The study of interactions that occur under direct and indirect discrimination strategies is the main theoretical contribution of our paper.

Some regulators enforce strict net neutrality, which bans any discrimination on either side. We refer to this type of neutrality as “full net neutrality.” Other regulators allow deviation on one side, e.g., paid prioritization on the content side or zero-rating on the user side; we term this situation “partial net neutrality.” For instance, California’s Net Neutrality Act of 2018 establishes an anti-discrimination principle for both the content and user sides by banning both zero-rating and sponsored data.1 Therefore, it goes one step further than the Federal Communications Commission’s Open Internet Order (Federal Communications Commission (2015)), which prohibits paid prioritization on the content side. Various countries other than the U.S. have similar regulations. India barred internet service providers (ISPs) from engaging in any form of discrimination, including zero-rating. Regulators in Canada, Chile, the Netherlands, and Slovenia have either extended net neutrality to cover zero-rating or expressed their concern over zero-rating considering that it nullifies the effects of net neutrality.

We raise several research questions that ensue from the interactions between one side and the

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other when discrimination is applied. From a welfare perspective, does partial net neutrality on either side dominate full net neutrality? Given partial net neutrality, is regulation of the content side or regulation of the user side more socially desirable? Under what conditions does partial net neutrality enhance welfare compared to no regulation? How do different regimes affect the ISP’s investment incentives?

To address the questions above, we consider a model in which a monopolistic platform mediates between two groups of internet users and two CPs. In the baseline model, we assume independent markets such that users in group $i \in \{1, 2\}$ only consume content provided by CP $i$ in a manner that can be described by linear demand functions. Both users and CPs benefit from indirect network effects such that more users bring greater utility for CPs by a common network externality parameter and vice versa. We consider heterogeneous CPs in that one CP requires greater bandwidth for content delivery, thereby incurring traffic management expenditures and a higher cost for the ISP. The deviations from net neutrality on either the content side or the user side or both sides are represented by price discrimination against the provider of more costly content (direct discrimination) and/or against the users of such content (indirect discrimination).

In Section 2, we first find that full net neutrality reduces welfare compared to no regulation due to consumption reallocation effects within sides and matching effects between sides. The sum of these effects within the content and the user side is positive because the ISP reallocates users and CPs from low markup content (with high cost) to high markup content (with low cost) when it is allowed to discriminate against both users and CPs. This situation not only improves total welfare within sides but also results in better matching between content and users, which in turn strengthens network externalities and further improves welfare.

Even if full net neutrality reduces welfare compared to full discrimination, this effect does not necessarily mean that partial net neutrality is also welfare-reducing. In this sense, Section 3 considers whether partial net neutrality on either side is welfare-enhancing or welfare-reducing compared to full net neutrality and no regulation. We first show that both forms of partial net neutrality enhance welfare compared to full net neutrality because of positive consumption reallocation effects within sides and positive matching effects between sides. We also show that positive matching effects imply that partial net neutrality on the user’s side results in greater welfare than no regulation if users benefit slightly more from participation on the content side than CPs benefit from participation on the user side.

In Section 4, we examine how the ISP’s optimal level of cost-reducing investment is affected by the different regimes. Given that the ISP’s incentive to invest depends on the quantity of content to which
the cost reduction applies, full net neutrality, which maximizes the quantity of content produced by high-cost CPs, results in the highest level of investment, whereas unconstrained discrimination discourages investment the most. For partial net neutrality regimes, regulation on the content side results in more investment than does regulation on the user side because the quantity of high-cost CP content sees a greater reduction when users are discriminated against. This finding demonstrates a potential conflict between static efficiency, which is achieved by choosing the appropriate allocation of consumption and maximization of the network externalities, and dynamic efficiency, which is achieved by maximizing investment.

In Section 5, we consider several model extensions. In Section 5.1, we relax the assumption on independent markets and examine the case in which users in group $i$ consume a mix of content from both high-cost and low-cost CPs. Sections 5.2 and 5.3 explore sources of deviations from net neutrality other than cost-driven motives. First, we focus on “own-content bias” arising from the vertical affiliation between the ISP and the CP. Additionally, we consider content popularity as another common source of deviations from net neutrality.

Finally, we provide concluding remarks in addition to policy implications in Section 6.

**Previous Literature** Substantial work has examined the static and dynamic effects of net neutrality in various settings. A strand of related literature investigates how banning paid prioritization affects investment incentives for the ISP and/or CPs (Krämer and Wiewiorra (2009); Choi and Kim (2010); Bourreau et al. (2015); Reggiani and Valletti (2016); Baake and Sudaric (2019); Baranes and Vuong (2020)). On the other hand, Somogyi (2017), Jullien and Sand-Zantman (2018), Schnurr and Wiewiorra (2018), and Maillé and Tuffin (2019) study how deviations from partial net neutrality on the user side, such as zero-rating and sponsored data programs, affect content consumption and welfare. Along the lines of our focus on a departure from net neutrality, i.e., price discrimination in a two-sided market, Choi et al. (2015) show that whether or not second-degree price discrimination, represented by paid prioritization, is beneficial depends on the CPs’ business models and how such models determine the allocation of total surplus between CPs and users.

Our paper differs from the above-mentioned works in that we focus on how imposing net neutrality on two different sides of the market leads the two sides to interact and on how this interaction affects welfare and investment incentives. In this regard, Gautier and Somogyi (2020), which investigates both paid prioritization and zero-rating (or sponsored data) to show how the ISP makes an optimal decision between these two practices, is the paper most closely related to ours. Gans (2015) is also in the same vein as ours given that he considers both *weak net neutrality* under which either the CP or user side is regulated and *strong net neutrality* under which the ISP sets uniform pricing for both CPs
and users. However, neither study focuses on the network externalities arising from the interaction between the content and user sides, and these externalities drive the main implications of our paper. To summarize, the existing literature on net neutrality focuses on the specificities of the discrimination strategies implemented by ISPs on the content and the user sides (typically, paid prioritization and zero-rating), whereas we place more emphasis on the fact that deviations from net neutrality are discrimination on either side or both sides of a two-sided market, triggered by heterogeneity on the content side.

Our paper is also related to the literature on price discrimination in general. Among others, Schmalensee (1981), Aguirre et al. (2010), and Cowan (2016) analyze the effects of monopolistic (third-degree) price discrimination on the consumer’s side in the traditional market setup; given one-sidedness, these papers focus on only direct price discrimination. Several studies look into diverse forms of platform discrimination in a two-sided market setup such as ours. Liu and Serfes (2013) examine how price discrimination in a one-sided market works differently from that in a two-sided market by showing that price discrimination, which is considered a competition-enhancing strategy in the former case, may soften competition in the latter. Kodera (2015) studies the effects of price discrimination on the advertiser’s side only, a practice that is conceptually the same as elimination of partial net neutrality on the content side. Lin (2020) shows that price discrimination on one side encourages media platforms to discriminate on the other. Jeon et al. (2021) analyze second-degree price discrimination in a two-sided market and find that banning price discrimination is optimal and welfare-enhancing if the incentive for one side (say, the user’s side) is in significant conflict with another side (say, the advertiser’s side). Lastly, Trégouët (2015) and Gomes and Pavan (2016) examine price discrimination involving both sides using a matching model. Our contribution to the literature on platform discrimination is to distinguish between direct and indirect price discrimination and make related policy recommendations.

2 (Full) Net Neutrality versus Unconstrained Discrimination

To model net neutrality on either or both the content side and the user side, we construct a model in which deviations from net neutrality are represented by price discrimination stemming from a higher cost induced by certain types of content. Absent any regulation, the providers of such higher-cost

\footnote{For example, if users on social networks value privacy protection, a high-quality service provided by platforms, represented by privacy-enhancing features, reduces profits for advertisers.}

\footnote{Other likely sources of price discrimination are popular content and content affiliated with ISPs, which are discussed in Sections 5.2 and 5.3.}
content would be discriminated against by ISPs, as would the internet users who consume such content. We first compare this situation with full net neutrality, under which the ISP charges uniform prices on both sides of the market. Then, in Section 3, we also consider unilateral discrimination on either the content side or the user side (i.e., partial net neutrality). This approach allows us to study how net neutrality regulations on both sides of the market interact with one another, which is the main contribution of our paper.

Our model consists of a monopolistic platform that mediates between two groups of internet users and two CPs, respectively. In the baseline model, we consider independent markets, that is, users in group $k \in \{1, 2\}$ only consume content provided by CP$_k$. The net utility of an internet user in group $k$ is given by $v_{uk} = \alpha_u q_{ck} - p_{uk}$, where $\alpha_u \in [0, 1)$ is a common network externality parameter, $q_{ck}$ is the quantity of content produced by CP$_k$, and $p_{uk}$ is the internet subscription fee charged by the ISP. Likewise, the net utility that CP$_k$ receives per unit of content is $v_{ck} = \alpha_c q_{uk} - p_{ck}$, where $\alpha_c \in [0, 1)$ is a common network externality parameter, $q_{uk}$ is the number of internet users in group $k$, and $p_{ck}$ is the fee charged by the ISP to make the content available to internet users.

The number of internet users in group $k$ is $q_{uk} = \Phi_u(v_{uk})$, where $\Phi_u' > 0$, and the quantity of content produced by CP$_k$ is $q_{ck} = \Phi_c(v_{ck})$, where $\Phi_c' > 0$. We suppose that $\Phi_u$ and $\Phi_c$ are linear functions. This leads us to focus on the reallocation effects of net neutrality regulations and to omit their potential effects on the total numbers of users and volumes of content. Without further loss of generality, to study the welfare effects of net neutrality regulation, we assume that $\Phi_u = 1 + b_u v_{uk}$ and $\Phi_c = 1 + v_{ck}$, where $b_u > 0$, which simply reflects that the demand functions on the content and the user sides are not necessarily symmetric. We also take the following regularity condition for granted: under all net neutrality regimes, the equilibrium price $p_{ik}$ increases with the corresponding marginal cost $c_{ik}$, $\forall i \in \{c, u\}$ and $\forall k \in \{1, 2\}$.

We suppose that the content produced by CP$_2$ yields extra costs for the ISP, such as network management expenditures, because it uses more bandwidth; for instance, CP$_2$ offers a high-definition video streaming service, whereas CP$_1$ is an online newspaper. Normalizing the marginal costs for users and the low-cost CP, CP$_1$, to zero, the profit of the ISP can be written as $\pi = p_{u1} q_{u1} + p_{u2} q_{u2} + p_{c1} q_{c1} + (p_{c2} - c) q_{c2}$, where $c \geq 0$ represents the extra cost induced by the content of CP$_2$.

We solve for the demand system as follows:

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4Imperfect substitution between content is discussed in Section 5.

5As shown in Appendix A, the stability condition is an upper bound on the network effects, namely, $4 - b_u (\alpha_c + \alpha_u)^2 > 0$.

6Cost asymmetries can be introduced on both sides of the market. However, they significantly increase the complexity of the analysis without providing additional insight. Overall, our findings extend to the case in which there is greater cost asymmetry on the content side than on the user side.
\[ q_{uk} = \Phi_u(\alpha_u q_{ck} - p_{uk}); \quad q_{ck} = \Phi_c(\alpha_c q_{uk} - p_{ck}), \]

which yields the demand functions \( q_{uk}(p_{ck}, p_{uk}) \) and \( q_{ck}(p_{ck}, p_{uk}) \). The regularity condition implies that \( \frac{\partial q_{uk}}{\partial p_{uk}} < \frac{\partial q_{uk}}{\partial p_{ck}} < 0 \) and \( \frac{\partial q_{ck}}{\partial p_{ck}} < \frac{\partial q_{ck}}{\partial p_{uk}} < 0 \). We then maximize the ISP’s profit with respect to prices. Under unconstrained discrimination, the equilibrium prices are such that \( \frac{\partial \pi}{\partial p_{uk}} = \frac{\partial \pi}{\partial p_{ck}} = 0 \), \( \forall k \in \{1, 2\} \).\(^7\) We obtain the following first order conditions under no regulation:

\[
\begin{align*}
q_{uk}(p_{ck}, p_{uk}) + p_{uk}(\partial q_{uk}/\partial p_{uk}) + (p_{ck} - c_{ck})(\partial q_{ck}/\partial p_{uk}) &= 0; \\
q_{ck}(p_{ck}, p_{uk}) + (p_{ck} - c_{ck})(\partial q_{ck}/\partial p_{ck}) + p_{uk}(\partial q_{uk}/\partial p_{ck}) &= 0,
\end{align*}
\]

where the marginal costs for CP\(_1\) and CP\(_2\) are \( c_{c1} = 0 \) and \( c_{c2} = c \), respectively. Under full net neutrality, i.e., when discrimination on either sides is banned, we maximize the ISP’s profit subject to the constraint \( p_{c1} = p_{c2} = p_c \). The resulting equilibrium prices are such that \( \frac{\partial \pi}{\partial p_u} = \frac{\partial \pi}{\partial p_c} = 0 \), \( \forall i \in \{1, 2\} \). We define \( Q_u = q_{u1} + q_{u2} \) and \( Q_c = q_{c1} + q_{c2} \), and obtain the following first order conditions under full net neutrality:\(^8\)

\[
\begin{align*}
Q_u(p_c, p_u) + p_u(\partial Q_u/\partial p_u) + (p_c - c/2)(\partial Q_c/\partial p_u) &= 0; \\
Q_c(p_c, p_u) + (p_c - c/2)(\partial Q_c/\partial p_c) + p_u(\partial Q_u/\partial p_c) &= 0.
\end{align*}
\]

Equations (2) and (3) show that prices are adjusted downward by the externalities across sides, which is a well-known principle in two-sided markets (e.g., Armstrong (2006)). This fact implies that the lower the price is on one side of the market, the higher the price is on the other side, which can be referred to as the **seesaw effect**.

We denote the equilibrium prices under unconstrained discrimination as \( p_{ik}^* \). The corresponding quantities are \( q_{ik}^* \) and welfare \( W^* \). Under full net neutrality, they are denoted as \( p_{ik}, \pi_{ik}, \) and \( W \). The welfare of group \( k \in \{1, 2\} \) on side \( i \in \{c, u\} \) is defined as follows:

\(^7\)The linearity assumption implies that the second-order conditions are satisfied.

\(^8\)As in Armstrong (2006), the platform’s profit can also be expressed as a function of utilities, as follows:

\[
\pi = (\alpha_u \Phi_u(v_{u1} - v_{u1})\Phi_u(v_{u1}) + (\alpha_u \Phi_u(v_{u2} - v_{u2})\Phi_u(v_{u2})
+ (\alpha_c \Phi_u(v_{c1} - v_{c1})\Phi_c(v_{c1}) + (\alpha_c \Phi_u(v_{c2} - v_{c2} - c)\Phi_c(v_{c2}).
\]

The equilibrium condition can then be determined by maximizing the profit of the ISP with respect to the utilities \( \{v_{c1}, v_{c2}, v_{u1}, v_{u2}\} \) as if the ISP were setting the latter directly. The first order conditions under no regulation are

\[
\Phi'_u(v_{u1})[(\alpha_c + \alpha_u)\Phi_u(v_{u1}) - v_{ui} - c_{ui}] - \Phi_c(v_{c1}) = 0; \quad \Phi'_u(v_{u2})[(\alpha_c + \alpha_u)\Phi_u(v_{u2}) - v_{ui} - c_{ui}] - \Phi_u(v_{u1}) = 0.
\]

Likewise, the first order conditions under full net neutrality are

\[
\Phi'_c(v_{c1})[(\alpha_c + \alpha_u)\Phi_u(v_{u1}) - v_{ci} - (c_{c1} + c_{c2})] - \Phi_c(v_{c1}) = 0; \quad \Phi'_u(v_{u1})[(\alpha_c + \alpha_u)\Phi_c(v_{c1}) - v_{ui} - c_{ui}] - \Phi_u(v_{u2}) = 0,
\]

where \( v_u = v_{u1} = v_{u2} \) and \( v_c = v_{c1} = v_{c2} \). This approach is particularly useful when attempting to generalize the study of full net neutrality to nonlinear demand.
\[ W_{ik} \equiv (p_{ik} - c)q_{ik} + \int_{v_{ik}^0}^{v_{ik}} \Phi_i(v)dv = q_{ik}(\alpha_iq_{-ik} - c_{ik}) - \int_{0}^{q_{ik}} \Phi_i^{-1}(q)dq, \]  

(4)

where \( v_{ik}^0 \) is such that \( \Phi_i(v_{ik}^0) = 0 \), that is, \( v_{u1}^0 = v_{u2}^0 = -1/b_u \) and \( v_{c1}^0 = v_{c2}^0 = -1 \). Total welfare is represented by \( W \equiv \sum_{ik} W_{ik} \). Allowing the ISP to deviate from full net neutrality results in a change in welfare \( \Delta W \equiv W^* - W \), that can be decomposed as follows:

\[ \Delta W = \left( q_{c1}^* - \bar{q}_c \right) \left[ p_{c1}^* - \left( p_{c2}^* - 2c \right) \right]/2 + \left( q_{u1}^* - \bar{q}_u \right) \left( p_{u1}^* - p_{u2}^* \right)/2 \]

Reallocation effect within the content side

\[ + \left( \alpha_u + \alpha_c \right) \left( q_{u1}^* q_{c1}^* + q_{u2}^* q_{c2}^* - 2\bar{q}_u \bar{q}_c \right)/2. \]

Matching effect between sides

(5)

Note that both the reallocation effects and the matching effect are reallocation effects: the former is the impact within sides of consumption reallocation, and the latter is that between sides of consumption reallocation. The following result ensues:\footnote{The proofs of all the lemmas and propositions are provided in Appendix B.}

**Lemma 1.** If the ISP is allowed to deviate from full net neutrality and discriminates on both sides of the market, the reallocation effect within the content side is positive. The reallocation effect within the user side is positive if \( \alpha_u > \alpha_c \) and negative otherwise; the matching effect between sides is positive.

Consumption reallocation effects are a well-known consequence of price discrimination. A critical difference between our model and the existing literature is that, in our model, the firm that discriminates is a platform, and reallocation effects are present on both sides of the market; these reallocation effects can work either in the same direction or in opposite directions depending on the relative intensity of the network effects.\footnote{In the literature, price discrimination also affects welfare through its impact on total output (e.g., Schmalensee (1981) or Cowan (2016)). This feature is not the case in our model because linear demand functions imply that discrimination leaves the total number of internet users and the total quantity of content unchanged.}

First, the reallocation effect is positive on the content side because the ISP charges higher fees to the provider of costlier content and lower fees to other CPs when it is allowed to discriminate. This setup reallocates consumption from lower-markup to higher-markup content, thereby improving welfare given that \( p_{c1}^* > p_{c2}^* - c > p_{c2}^* - 2c \). Similar effects have been studied in standard one-sided markets and are discussed in the literature on cost-based discrimination (e.g., Chen and Schwartz (2015)).

Unlike the reallocation effect within the content side, the reallocation effect within the user side is positive if and only if users benefit more from an extra unit of content than CPs benefit from
an extra user, i.e., when $\alpha_u > \alpha_c$. This fact is explained by the fact that when the ISP is allowed to discriminate, it always assigns a higher utility to users who consume less costly content, while it charges them a higher subscription fee if and only if $\alpha_u > \alpha_c$; thus, $q_{u1}^* - \bar q_u$ is always positive, and $p_{u1}^* - p_{u2}^* > 0$ if and only if $\alpha_u > \alpha_c$.

Specifically, whether users in group 1 are charged a higher or a lower price when the ISP is allowed to discriminate depends on the balance of network effects. If discrimination is allowed on the user side only, the subscription fee charged to users in group 1 would decrease relative to the fee charged under net neutrality conditions because the content those users consume induces lower costs for the ISP. By symmetry, users in group 2 would be charged a higher subscription fee. However, since discrimination is also allowed on the content side, the situation results in lower fees for CP$_1$, which, in turn, allows the ISP to raise the subscription fee $p_{u1}$, according to the seesaw effect. Similarly, discrimination on the content side reduces the subscription fee $p_{u2}$. In this regard, we find that if $\alpha_u > \alpha_c$, then $p_{u1}^* - p_{u2}^* > 0$; otherwise, $p_{u1}^* - p_{u2}^* \leq 0$. This pattern implies that the relative size of the network effects determines how the internet subscription fees are affected by the effects of discrimination on the user or content side.

Finally, a positive matching effect arises from the fact that discrimination affects the distribution of users and content. The even distribution of users and content induced by net neutrality, i.e., uniform prices on both sides, is the worst possible allocation from a matching perspective. Indeed, each group of users then consumes only 50% of the total amount of content, and the content of each content provider is consumed by only 50% of the internet users. By contrast, the best allocation in terms of matching occurs when all users belong to group $i$ and all content is produced by CP$_i$. Then, 100% of the content is consumed by 100% of the users. By assigning more weights to users in group 1 and content by CP$_1$, discrimination results in better matching of users and content than does net neutrality. The total welfare impact of net neutrality depends on the sum of the effects described above and can be summarized as follows.

**Proposition 1.** *Full net neutrality reduces welfare compared to unconstrained discrimination.*

Proposition 1 is explained by the fact that the total reallocation effect within the content and user side is always positive. This fact follows from the regularity condition: the positive reallocation effect within the content side dominates the potential negative reallocation effect within the user side because the former is triggered directly by cost asymmetries across CPs, whereas the latter is the indirect consequence of users’ preferences in terms of content.

In contrast to the existing literature on monopolistic, third-degree price discrimination, the ex-
istence of a matching effect implies that in two-sided markets welfare can increase even if the total output is constant and the total reallocation effect within sides is negative.\footnote{A fundamental result in the early literature on monopolistic, third-degree price discrimination is that a positive output effect is a necessary condition for price discrimination to improve welfare—see, in particular, Schmalensee (1981), Varian (1985) and Schwartz (1990). This outcome remains a central result on which recent contributions are built—see, e.g., Cowan (2016). This condition has also been discussed in the context of discrimination across groups of users with interdependent demands—see, in particular, Layson (1998).} For example, suppose that in our model the ISP discriminates against CP\(_1\) and users in group 1. Although this situation would result in worse allocation of users and content within sides, welfare could still improve because the reallocation of users from group 1 to group 2 and of content from CP\(_1\) to CP\(_2\) would entail improved matching between sides.\footnote{See also the case of affiliated content discussed in Section 5.2.}

### 3 Full vs. Partial Net Neutrality

As shown in Section 2, discrimination on both sides of the market can result in a negative reallocation effect within the user side compared to net neutrality. This raises the question of whether banning discrimination and enforcing net neutrality on the user side only, i.e., authorizing discrimination on the content side only, may be socially optimal. Section 3 allows us to assess this argument by comparing the welfare effects of imposing net neutrality on either side or both sides of the market.

We derive the two equilibria for the conditions in which discrimination is permitted on only one side of the market and compare them with the conditions in which there is full net neutrality and full discrimination. In the case of net neutrality on the content side, we maximize the ISP’s profit with respect to prices and subject to the constraint \(p_{c1} = p_{c2} = p_c\). Likewise, in the case of net neutrality on the user side, we maximize it subject to \(p_{u1} = p_{u2} = p_u\).

We use the subscripts \(NC\) and \(NU\) to refer to net neutrality on the content side and on the user side, respectively. Full net neutrality and full (unconstrained) discrimination are referred to as \(N\) and \(D\), respectively. When comparing two net neutrality regimes, we denote the variation in welfare as \(\Delta W = W^* - \overline{W}\), where \(W^*\) and \(\overline{W}\) are the welfare levels in the less binding and more binding regimes, respectively. For example, if we study the welfare effect of full discrimination compared to that of partial net neutrality on the user side, we have \(W^* \equiv W^D\) and \(\overline{W} \equiv W^{NU}\). When comparing the two partial net neutrality regimes, we write, by convention, \(W^* \equiv W^{NU}\) and \(\overline{W} \equiv W^{NC}\). We use similar notations for prices and quantities. Then, for all welfare comparisons, we obtain the following:
\[ \Delta W = \frac{(q_{i^*c} - \bar{q}_c)(\bar{p}_{i^*c} - (\bar{p}_{c^2} - c))}{\text{Realloc. effect within the content side}} + \frac{(q_{u^*1} - \bar{q}_u)(\bar{p}_{u^*1} - \bar{p}_{u^*2})}{\text{Realloc. effect within the user side}}\]

\[\left[ (\alpha_c + \alpha_u) \sum_i(q_{i^*c}q_{c^2}^* - \bar{q}_{i^*c}\bar{q}_{c^2}) - (\alpha_c - \alpha_u) \sum_i(\bar{q}_{i^*c}q_{c^2}^* - q_{i^*c}\bar{q}_{c^2}) \right] / 2, \]

where \( \tilde{p}_{ik} \equiv (p_{ik}^* + \bar{p}_{ik})/2, \forall i \in \{c, u\} \) and \( k \in \{1, 2\} \), which nests the decomposition of Section 2. The comparison of net neutrality regimes yields the following result:

**Proposition 2.** The ranking of the net neutrality regimes in terms of welfare, which depends on the relative size of the two network externalities (i.e., \( \alpha_u \) and \( \alpha_c \)), is (i) \( W^N \leq W^{NC} \leq W^{NU} \leq W^D \), (ii) \( W^N \leq W^{NC} \leq W^D \leq W^{NU} \), or (iii) \( W^N \leq W^{NU} \leq W^{NC} \leq W^D \).

From a policy perspective, the most important aspect of Proposition 2 is that partial net neutrality on the user side can dominate unconstrained full discrimination. The basic effects of full discrimination compared to those of partial net neutrality on the user side are an increase in the subscription fee paid by users in group 2 and a decrease in the fee paid by users in group 1, i.e., \( p_{u1}^D < p_{u1}^{NU} = p_{u2}^{NU} < p_{u2}^D \). Because of the seesaw effect, these effects, in turn, reduce the price gap on the content side, i.e., \( p_{c1}^{NU} < p_{c1}^D = p_{c2}^D < p_{c2}^{NU} \). Since \( q_{ic} = q_{ic}(p_{ic}, p_{iu}) \), where \( \partial q_{ic}/\partial p_{ic} < \partial q_{ic}/\partial p_{iu} < 0 \), such price variations on both sides of the market work to increase the gap in the quantities produced by CP1 and CP2, i.e., \( q_{c2}^D < q_{c2}^{NU} = q_{c1}^{NU} < q_{c1}^D \).

Here, we find conflicting effects on the distribution of users because \( q_{iu} = q_{iu}(p_{ic}, p_{iu}) \), where \( \partial q_{iu}/\partial p_{iu} < \partial q_{iu}/\partial p_{ic} < 0 \). On the one hand, the greater dispersion of internet subscription fees (i.e., \( p_{u1}^D \) and \( p_{u2}^D \)) exacerbates the imbalance in the sizes of the user groups. On the other hand, the lower dispersion of the fees paid by the CPs (i.e., \( p_{c1}^D \) and \( p_{c2}^D \)) works in the opposite direction. The former effect dominates if the CPs benefit more from an extra user than users benefit from an additional unit of content: that is, if \( \alpha_u \leq \alpha_c \), then \( q_{u2}^D < q_{u2}^{NU} = q_{u1}^{NU} < q_{u1}^D \), meaning that there is both a higher percentage of users in group 1 and a higher percentage of content produced by CP1 under full discrimination. Otherwise, that is, if \( \alpha_u > \alpha_c \), then \( q_{u2}^{NU} < q_{u2}^D < q_{u1}^D < q_{u1}^{NU} \), the latter effect dominates.

These variations in quantities and prices result in a positive total reallocation effect within sides similar to that described in Section 2. The impact on matching between sides and the resulting sign of the variation in network externalities are more ambiguous. If \( \alpha_u < \alpha_c \), discrimination on both sides increases not only the number of users in group 1 but also the quantity of content produced by CP1 due to the direction of the network effects. This results in better matching between users and content,
making full discrimination more likely to be welfare-enhancing. However, if $\alpha_u > \alpha_c$, the size of the first group of users increases, whereas the quantity of content produced by CP\textsubscript{1} decreases. When the imbalance in the network effects is not too much in favor of $\alpha_u$, this results in worse matching between users and content, which in turn dominates the positive reallocation effect. Then, net neutrality on the user side yields higher welfare than unconstrained discrimination, as shown in region (ii) in Figure 1.

Proposition 2 also shows that both types of partial net neutrality dominate full net neutrality. Under partial net neutrality on the user side, the ISP discriminates against CP\textsubscript{2}, that is, against the CP that produces more expensive content. Likewise, under partial net neutrality on the content side, the ISP discriminates against users in group 2, that is, against users who consume more expensive content. In both cases, the reallocation effects within sides are positive, and the matching between sides is better than under conditions of full net neutrality.

In a comparison of the two partial net neutrality regimes, net neutrality on the user side usually results in higher welfare. However, when the imbalance in network effects is strongly in favor of $\alpha_c$, there exists a range of parameter values such that partial net neutrality on the content side dominates partial net neutrality on the user side, as illustrated in region (iii) in Figure 1.

Finally, partial net neutrality on the content never improves welfare compared to unconstrained discrimination because it results in an inferior allocation of consumption within sides and worse matching.
between sides compared to unconstrained discrimination.\textsuperscript{13}

Concerning the welfare effect of partial net neutrality on the user side, Jeitschko et al. (2021) show that allowing zero-rating is socially desirable because it encourages users to consume more content than they would under full net neutrality. Our findings show similar results but go one step further by comparing zero-rating with another type of partial net neutrality, paid prioritization: depending on the size of the network effects on each side, allowing zero-rating can be more welfare-enhancing or welfare-reducing than allowing paid prioritization. In this regard, Gautier and Somogyi (2020) show that whether prioritization or zero-rating benefits consumers depends on the degree of network congestion, which is an important consideration, especially in the long run. We consider another important but distinct aspect of the regulation, which is the interaction between CPs and users via consumption reallocation effects. This important difference arises from the network externalities that we take into account in this paper.

4 Full or Partial Net Neutrality and the ISP’s Investment Incentives

We now turn to the impact of net neutrality on either or both sides of the market on the ISP’s incentives to invest, a key aspect of the debate on net neutrality. Indeed, opponents of net neutrality often argue that net neutrality deters infrastructure investment by not rewarding ISPs (e.g., Yoo (2008); Katz (2017)). Several other studies also show that the discriminatory regime induces more investment in a broadband capacity due to either less scarce network capacity under the discriminatory regime or the ISP’s ability to extract rents from CPs in the form of priority fees (e.g., Bourreau et al. (2015); Baake and Sudaric (2019)).

To examine this argument, we consider that the ISP incurs a fixed cost for infrastructure investment that reduces the extra marginal cost $c$ induced by the content of CP\textsubscript{2}. In particular, we write the net profit of the ISP as $\Pi(x) = \pi(x) - f(x)$, where $k - x \equiv c$, $\pi(x) = p_{u1}q_{u1} + p_{u2}q_{u2} + p_{c1}q_{c1} + (p_{c2} - k + x)q_{c2}$, as in Sections 2 and 3, and $x$ is the reduction in marginal cost $c$ corresponding to the fixed cost $f(x)$. The fixed cost increases in the cost reduction, and we assume that it is sufficiently convex to ensure that the second-order condition $d^2\Pi(x)/dx^2 < 0$ is satisfied. The equilibrium level of investment is then such that the marginal investment cost, $df/dx$, equals the marginal profit resulting from an

\textsuperscript{13}For the impact of net neutrality regulation on the ISP profit, the revealed preferences indicate that the ISP is better off when discrimination is allowed on either side or both sides than it is under full net neutrality. Likewise, from the point of view of the ISP, unconstrained discrimination dominates net neutrality on either side. The only case in which the sign is a priori indeterminate is the comparison between net neutrality on the content side and net neutrality on the user side, but it is only a matter of computation to show that the latter yields higher profits (see Appendix B). This result can be explained simply by the standard regularity condition, according to which direct effects on one side dominate induced effects on the other.
incremental cost reduction, \( d\pi/dx \), which can be referred to as the incentive to invest. The following result ensues:

**Proposition 3.** The ranking of the incentives to invest is

\[
d\pi^D/dx < d\pi^{NU}/dx < d\pi^{NC}/dx < d\pi^N/dx.
\]

The level of the incentive to invest depends simply on the quantity of content to which the cost reduction applies. Therefore, full net neutrality results in the highest level of investment because it maximizes the quantity of content produced by CP2. Intuitively, this result leads to net neutrality on the content side, net neutrality on the user side, and unconstrained discrimination.

Proposition 3, which shows that net neutrality encourages investment because it increases the market share of the higher-cost content, contradicts the findings of some previous studies on the effects of (partial) net neutrality on capacity-expanding investment incentives. In particular, Baake and Sudaric (2019) show that investment incentives are the highest under the discriminatory regime, compared to net neutrality, given limited network capacity. Similarly, Bourreau et al. (2015) show that no regulation, which allows ISPs to extract extra rents from CPs via priority fees, results in more investment in network capacity and content innovation than does net neutrality. That is, considering network congestion issues, both studies find that (partial) net neutrality, which leads to greater content consumption, brings too much traffic, lowers capacity investment, and thereby decreases overall welfare. In our model, net neutrality also results in network congestion, which is modeled implicitly by higher marginals costs. However, the content-side departure from net neutrality we study is closer to congestion penalties than paid prioritization. This explains the contrast between our finding and that of the above-mentioned papers.

Proposition 3 suggests that there exists a contradiction between, on the one hand, static efficiency, achieved through the appropriate allocation of consumption and, on the other hand, dynamic efficiency, achieved through the maximization of investment.\(^{14}\)

5 **Extensions and Discussion**

In this section, we present several extensions of our baseline model. We first consider internet users who consume a mix of content from both CPs. In particular, we assume that users in group \( i \) consume

\(^{14}\)Since investment benefits not only the ISP but also the CPs and internet users, the private level of investment is always lower than the socially optimal level, and more investment always improves welfare. Additionally, note that this line of reasoning omits the dynamic of content production and the fact that CPs are more likely to limit their bandwidth consumption if they are discriminated against when they cause network congestion.
a fixed share $\beta \in (1/2, 1]$ of the content produced by CP$_{i}$ and a fixed share $1 - \beta$ of the content produced by the other CP. We also consider new sources of deviations from net neutrality. First, we study the so-called “own-content bias” by assuming that the ISP owns a non-controlling share $\gamma$ of CP$_{2}$. We then analyze the case of discrimination triggered by popular content. We represent the latter as an upward shift in the demand for CP$_{2}$, which is then assumed to have an initial mass of users $\delta \geq 1$. These extensions are illustrated in Figure 2.

Figure 2: Net neutrality and welfare: baseline model and extensions (where $b_u = 3/2$ and $c = 1/4$)

### 5.1 Mix of Content

Internet users usually consume a mix of content from a variety of content providers. A simple way to incorporate this fact into our model is to assume that users in group $i$ consume a share $\beta \in (1/2, 1]$
of the content produced by CP \(_i\) and a share \(1 - \beta\) of the content produced by the other CP. Then, \(\forall i \in \{1, 2\}\) and \(i \neq j\), the demand function of the internet users becomes \(q_{ui} = 1 + b_u[\alpha_u(\beta q_{ci} + (1 - \beta)q_{cj}) - p_{ui}]\), and that of the content providers is \(q_{ci} = 1 + \alpha_c[\beta q_{ui} + (1 - \beta)q_{uj}] - p_{ci}\). The baseline model described in Sections 2 and 3 corresponds to the case in which \(\beta = 1\); the lower \(\beta\) is, the more similar internet users are in terms of the mix of content they consume.

The decomposition of the welfare effects becomes

\[
\Delta W = (q_{c1}^* - \bar{q}_c)\left[\bar{p}_{c1} - (\bar{p}_{c2} - c)\right] + (q_{u1}^* - \bar{q}_u)\left[p_{u1} - \bar{p}_{u2}\right] + \left[(\alpha_c + \alpha_u)R - (\alpha_c - \alpha_u)S\right]/2,
\]

where \(R = \beta \sum_i (q_{ui}^* - \bar{q}_ui) + (1 - \beta) \sum_{i \neq j} (q_{ui}^* q_{cj}^* - \bar{q}_ui \bar{q}_{cj})\) and \(S = \beta \sum_i (\bar{q}_ui q_{ci}^* - q_{ui}^* \bar{q}_{ci}) + (1 - \beta) \sum_{i \neq k} (\bar{q}_ui q_{cj}^* - q_{ui}^* \bar{q}_{cj})\). Overall, the same mechanism is at play as in the baseline model, and we obtain the following result:

**Lemma 2.** If users from the two groups consume a mix of content, the ranking of the net neutrality regimes in terms of welfare remains \(W^N \leq W^{NC} \leq W^{NU} \leq W^D\), \(W^N \leq W^{NC} \leq W^D \leq W^{NU}\), or \(W^N \leq W^{NU} \leq W^{NC} \leq W^D\).

However, the extension shows that the more similar the groups of users are to each other, the more likely it is that partial net neutrality on the user side dominates full discrimination, as shown in Figure 2, panels 2a and 2b. This is explained by the fact that the former is more likely to improve the matching between users and content than in the baseline model. Compared to full discrimination, partial net neutrality on the user side reduces asymmetry in the quantity of content produced by CP\(_1\) and CP\(_2\) but increases asymmetry in the size of the user groups, as in Section 3. The former effect weakens the positive matching effect, whereas the latter effect strengthens it. Increased symmetry across user groups affects the balance of these two effects and makes the latter effect more prevalent.\(^{15}\)

By contrast, the fact that internet users consume a mix of content makes partial net neutrality on the content side less likely to dominate partial net neutrality on the user side. Indeed, partial net neutrality on the content side then yields similar outcomes as full net neutrality, resulting, as shown in Section 3, in the worst allocation of users and content among the four regulatory regimes.

\(^{15}\)Formally, \(q_{c1}^{D} < q_{c2}^{NU} < q_{c1}^{NU} < q_{c2}^{D}\) and, if \(\alpha_u > \alpha_c\), \(q_{u2}^{NU} < q_{u2}^{D} < q_{u1}^{D} < q_{u1}^{NU}\). The range of parameter values such that \(W^{NU} > W^{D}\) increases when \(\beta\) decreases because, \(\forall \alpha_u > \alpha_c\), \(\partial(q_{c1}^D - q_{c1}^{NU})/\partial \beta = \partial(q_{u2}^{NU} - q_{u2}^{D})/\partial \beta > 0\) and \(\partial(q_{u1}^{NU} - q_{u1}^{D})/\partial \beta < 0\).
5.2 Affiliated Content

Thus far, we have considered cost as the single source of deviations from net neutrality. Another commonly observed source of discrimination across CPs and, indirectly, internet users is the fact that ISPs may produce their own content or may own shares in CPs. In particular, ISPs may discriminate in favor of such CPs, a situation that can be referred to as an “own-content bias.”

Suppose that the ISP owns a noncontrolling share $\gamma \geq 0$ of CP$_2$. Then, considering that CPs and users are otherwise symmetric, the ISP’s profit becomes

$$\pi = p_{u1}q_{u1} + p_{u2}q_{u2} + p_{c1}q_{c1} + p_{c2}q_{c2} + \gamma \int_{v_{u2}^0}^{v_{u2}(q_{c2}(p_{c2}, p_{u2}))} \Phi_u(v)dv.$$  \hspace{1cm} (8)

We maximize this profit with respect to prices, subject to the constraints of the four net neutrality regimes, and compare the equilibria. The results are not as tractable as those obtained in the baseline model, but numerical simulations indicate that our main findings remain valid, as illustrated in Figure 2c. Although the overall welfare impact of net neutrality regulation is similar to that in the baseline model, the underlying effects differ. For example, compared to full net neutrality, unconstrained discrimination yields a negative reallocation effect within sides. Indeed, content is reallocated from CP$_1$ to CP$_2$ because the fee paid by the latter decreases (own-content bias). This, in turn, results in a more asymmetric allocation of content, which reduces welfare.

The induced reallocation effect within the user side is negative if $\alpha_c > \alpha_u$ and positive otherwise. Nevertheless, from the regularity condition, the direct effect on the content side always dominates the induced effect on the user side, and the total reallocation effect within sides is negative. On the other hand, the asymmetric allocation of content induced by the own-content bias tends to result in a better matching between users and content. This positive effect dominates the negative reallocation effect within sides, and unconstrained discrimination dominates net neutrality.

Overall, the symmetric allocation of content and users induced by net neutrality and the resulting poor matching between users and content explain why welfare is lower in this regime than when discrimination is allowed on either or both sides of the market. As in Section 3, the exact ranking of the net neutrality regimes is determined by the balance of network effects, which determines the direction and intensity of the reallocation effects within and between sides.
5.3 Popular Content

Content popularity is another common source of deviations from net neutrality. This can be modeled by assuming that CP$_2$ is in higher demand than CP$_1$. In particular, suppose that $\Phi_{c1} = 1 + b_c(n_c q_{u1} - p_{c1})$ and $\Phi_{c2} = \delta + b_c(n_c q_{u2} - p_{c2})$, where $\delta \geq 1$, and that the CP and internet users are otherwise symmetric. Then, we obtain the following result.

Proposition 4. If deviations from net neutrality originate in content popularity, then the ranking of the net neutrality regimes in terms of welfare is $W^D \leq W^{NU} \leq W^{NC} \leq W^N$, $W^{NU} \leq W^D \leq W^{NC} \leq W^N$, or $W^{NU} \leq W^{NC} \leq W^D \leq W^N$.

Unlike discrimination based on costs and shareholding, net neutrality is the best regime in terms of welfare when discrimination ensues from content popularity. As shown in the literature on discrimination in one-sided markets, demand-based discrimination entails a negative reallocation effect because consumers with higher demand are charged higher prices. Likewise, in our model, any deviation from net neutrality (on either or both sides) results in a negative total reallocation effect within sides. If discrimination is allowed on the content side only, the ISP discriminates against the provider of popular content, that is, CP$_2$. If discrimination is allowed on the user side only, the ISP discriminates against the users of popular content, that is, users in group 2. Finally, if discrimination is allowed on both sides, the ISP discriminates against CP$_2$, and the resulting negative reallocation effect within the content side dominates regardless of which users are discriminated against.

In addition, since content of CP$_2$ is in higher demand than that of CP$_1$, all other thing held constant, reallocation of users towards group 1 and content towards CP$_1$ may reduce the network externalities and therefore welfare. Combined with the reallocation effects within sides, this explains why full net neutrality dominates unconstrained discrimination and partial net neutrality.

Given the negative reallocation effect within the content side and the regularity condition, the result that partial net neutrality on the content side always dominates partial net neutrality on the user side is rather intuitive. A more surprising result is that unconstrained discrimination may dominate net neutrality on the content side. As illustrated in Figure 2d, this occurs only if demand asymmetry is limited and $\alpha_c$ is much higher than $\alpha_u$, that is, if content providers benefit from extra users much more than users benefit from extra content providers. Then, allowing discrimination on the content side increases network externalities, which offsets the negative reallocation effects, which are limited for this range of parameter values.

The welfare effects of net neutrality regulations depend on the sources of discrimination among CPs and internet users. Table 1 summarizes these differences. Full net neutrality is always the best regime.
when discrimination is triggered by popular content, whereas either unconstrained discrimination (i.e., no regulation) or net neutrality on the user side (e.g., allowing paid prioritization while regulating zero-rating) maximizes social welfare when discrimination is based on costs or affiliated content. This mirrors the literature on third-degree price discrimination in one-sided markets, where demand-based discrimination yields negative consumption reallocation effects and reduces welfare unless total output increases (Schmalensee (1981)) and where cost-based discrimination (or “differential pricing”) yields positive consumption reallocation effects if the markup is greater in the lower-cost market than in the higher-cost market and thereby improves welfare unless total output decreases (Chen and Schwarts (2015)).

6 Conclusion

We studied how net neutrality regulation on the content and user sides affects welfare through network effects between the two sides. We developed a model in which, absent any regulation, a monopolistic ISP would discriminate against high-cost content and users of such content. In this framework, we found that partial net neutrality on the user side is the optimal regime if users benefit from an extra unit of content slightly more than CPs benefit from an extra user. Otherwise, no regulation, i.e., allowing price discrimination on both the user and content sides, yields higher welfare. Our results also illustrate that the side where partial net neutrality is implemented matters. In most cases, partial net neutrality is more desirable (or less harmful) on the user side than the content side. In other words, compared to uniform prices on both sides, direct price discrimination on the content side is usually more welfare-enhancing than indirect price discrimination on the user side. However, if the CP values an additional user far more than a user values an additional CP, partial net neutrality on the content side is more welfare-improving than its user side counterpart. Thus, our analysis indicates that the balance of cross-group network effects is a key determinant of the socially optimal net neutrality regime.

The welfare results have several important policy implications. First, strict net neutrality, such as California SB822, which regulates both the content and user sides, does not necessarily improve welfare. In our model, full net neutrality, which appears to benefit consumers by leveling the playing field in the content market, reduces welfare more than does partial net neutrality or no regulation. Additionally, as emphasized earlier, whether partial net neutrality on one side is welfare-enhancing or welfare-reducing, compared to either net neutrality on the other side or no regulation depends on the relative size of the network effects. Thus, a close analysis of cross-group network externalities is
required prior to implementing a specific policy remedy.

Our result regarding the investment incentives for the ISP under different regulatory regimes shows the existence of a tradeoff between static efficiency, achieved under unconstrained discrimination or net neutrality on the content side, which optimizes consumption allocation within sides and matching between users and content providers, and investment incentives, maximized under full net neutrality under which higher-cost content has the greater market share.

In the main analysis, we consider cost-based discrimination in the situation where one CP’s content incurs a higher delivery cost than that of the other. However, many different issues may motivate content discrimination, and we find that the welfare effects of net neutrality regulations depend on the source of discrimination across CPs and users. For instance, full net neutrality is always the best regime when discrimination is triggered by content popularity, whereas either no regulation or partial net neutrality on the user side maximizes welfare when discrimination is driven by cost asymmetry or by vertical affiliation between the ISP and a CP. Therefore, investigating the sources of discrimination among CPs and internet users is required prior to designing specific net neutrality regulations.

We conclude by emphasizing that the results presented in our paper have broader implications for the effects of a ban on price discrimination in two-sided markets. One such example is neutrality of e-commerce platforms, which Fang and Kim (2021) consider in the form of a ban discrimination among downstream sellers (tiered pricing for data provision) and/or discrimination among consumers (differentiated targeted advertising). The main findings of our study, which can be generalized to other contexts, can help policymakers design more effective regulation of discrimination in two-sided markets.

References


Appendix

A Baseline model: cost-based discrimination

The profit of the ISP is $\pi = p_{u1} q_{u1} + p_{u2} q_{u2} + p_{c1} q_{c1} + (p_{c2} - c) q_{c2}$, which can be written as follows.

$$\pi(v) = (\alpha_u \Phi_c(v_{c1}) - v_{u1})\Phi_u(v_{u1}) + (\alpha_u \Phi_c(v_{c2}) - v_{u2})\Phi_u(v_{u2}) + \pi(v) = (\alpha_u \Phi_c(v_{c1}) - v_{u1})\Phi_u(v_{u1}) + (\alpha_u \Phi_c(v_{c2}) - v_{u2})\Phi_u(v_{u2}),$$

or

$$\pi(p) = p_{c1} q_{c1}(p_{c1}, p_{u1}) + p_{c2} q_{c2}(p_{c2}, p_{u2}) + p_{u1} q_{u1}(p_{u1}, p_{c1}) + (p_{u2} - c) q_{u2}(p_{u2}, p_{c2}),$$

where $q_{c1}(p_{ci}, p_{ui}) = [1 - p_{ci} + \alpha_c(1 - b_u p_{ui})]/J$, $q_{ui}(p_{ui}, p_{ci}) = [1 - b_u p_{ui} + \alpha_u(1 - p_{ci}) b_u]/J$, and $J = 1 - b_u \alpha_c \alpha_u$. We also define $L = 4 - b_u (\alpha_c + \alpha_u)^2$. The regularity condition is $L > 0$, which implies $J > 0$. Finally, given the regularity condition, all prices are nonnegative if and only if $2 + \alpha_u - \alpha_u - b_u(\alpha_u^2 + \alpha_u) > 0$ and $8 - [c \alpha_c L + 4(1 + \alpha_c + \alpha_u) \alpha_c - 4 \alpha_u - 2(\alpha_c - \alpha_u)c] b_u > 0$.

The equilibrium prices under unconstrained discrimination can be obtained by maximizing either $\pi(v)$ with respect to the vector of utilities $v$ or $\pi(p)$ with respect to the vector of prices $p$. The equilibrium prices under full net neutrality can also be derived in either of these ways, but maximization is subject to the constraint $v_{c1} = v_{c2}, v_{u1} = v_{u2}$ in the former case and to the constraint $p_{c1} = p_{c2}, p_{u1} = p_{u2}$ in the latter case. We find the following:

$$p_{c1} = (2 + \alpha_c - \alpha_u - \alpha_u \alpha_u b_u - \alpha_u^2 b_u)/(4J);$$
$$p_{c2} = [2 + \alpha_c - \alpha_u + 2c - \alpha_u^2 b_u - \alpha_c \alpha_u (1 + c) b_u]/(4J);$$
$$p_{u1} = [2/b_u + \alpha_u - \alpha_c (1 + \alpha_u) - \alpha_c^2]/(4J);$$
$$p_{u2} = [2/b_u + (\alpha_u - \alpha_c) (1 - c) - \alpha_c^2 - \alpha_c \alpha_u]/(4J);$$
$$p_c^N = p_c^{NC} = (p_{c1} + p_{c2})/2, p_u^N = p_u^{NU} = (p_{u1} + p_{u2})/2;$$
$$p_c^{NU} = p_c^N - c/4; p_u^{NC} = p_u^N - c \alpha_c/4.$$

We use these equilibrium prices to determine the equilibrium quantities and welfare.

B Omitted Proofs

Proof of Lemma 1. The reallocation effect within the content side induced by unconstrained discrimination compared to net neutrality is given as follows:
\[ (q_c^* - q_c)(p_u^* - p_u^c) = c^2 (1 + J + L)/(32J^2) > 0. \]  

(12)

The corresponding reallocation effect within the user side is given as follows:

\[ (q_u^* - q_u)(p_u^* - p_u^c)/2 = b_u(\alpha_u - \alpha_c)(\alpha_u + \alpha_c)c^2/(64J^2), \]

which is positive if and only if \( \alpha_u > \alpha_c \). Finally, the variation in network externalities is

\[ (\alpha_u + \alpha_c)(q_u^*q_c^* + q_u^*q_c^2 - 2\overline{q}_u\overline{q}_c)/2 = b_u(\alpha_u + \alpha_c)c^2/(32J^2) > 0. \]

(14)

**Proof of Proposition 1.** The sum of the effects mentioned above yields the following:

\[ W^D - W^N = c^2 (8 + L)/(64J^2) > 0. \]

(15)

**Proof of Proposition 2.** The total welfare effect of full discrimination compared to net neutrality on the user side is given as follows:

\[ W^D - W^{NU} = [(c - \alpha_u)c^2b_u]M/(16J^2L^2), \]

(16)

where \( M = 2\alpha_c^2\alpha_u b_u^2 - \alpha_c^2\alpha_u b_u(23 - 5\alpha_u^2b_u) - \alpha_u^3b_u(3 - 5\alpha_u^2b_u) - \alpha_u(4 + 5\alpha_u^2b_u - \alpha_u^4b_u^2) + \alpha_c(20 - \alpha_u^2b_u + 3\alpha_u^4b_u^2) \). This is negative if and only if \( \alpha_u \in [\alpha_c, \tilde{\alpha}_u] \), where \( \tilde{\alpha}_u \) is such that \( M = 0 \). The total welfare effect of net neutrality on the user side compared to that on the content side is given as follows:

\[ W^{NU} - W^{NC} = c^2(3 - 3\alpha_c^2b_u - 2\alpha_c\alpha_u b_u + \alpha_u^2b_u - \alpha_c^4b_u + 2\alpha_c^3\alpha_u b_u^2)/L^2, \]

(17)

which is negative if \( \alpha_c - \alpha_u^2b_u - N < \alpha_u < \alpha_c - \alpha_u^2b_u + N \) where \( N = \sqrt{b_u(-3 + 4\alpha_c^2b_u - \alpha_c^4b_u^2 + \alpha_c^6b_u^4)}/b_u \) and positive otherwise.

Additionally, we find the following:

\[ W^{NC} - W^N = ac^2c^2b_u(1 + 2J + ac^2b_u)/L^2 > 0; \]

\[ W^{NU} - W^N = c^2(1 + 2J + \alpha_u^2b_u)/L^2 > 0; \]

\[ W^D - W^{NC} = c^2[48 - 4(13\alpha_c^2 + 26\alpha_c\alpha_u + \alpha_u^2)b_u + 8\alpha_c(\alpha_c + \alpha_u)(\alpha_c^2 + 10\alpha_c\alpha_u + \alpha_u^2)b_u^2 + \alpha_u^2(\alpha_c + \alpha_u)^2(5\alpha_c^2 - 22\alpha_c\alpha_u - 7\alpha_u^2)b_u^3 - \alpha_c^3(\alpha_c - 2\alpha_u)(\alpha_c + \alpha_u)^4b_u^4]/(J^2L^2), \]

(18)

which is positive as long as the regularity condition is satisfied and the prices are non-negative. This completes the proof of Proposition 2.

Revealed preferences indicate that the ISP is better off when discrimination is allowed on either side or both sides than it is under full net neutrality. For similar reasons, ISP profits are higher
under unconstrained discrimination than under net neutrality on either side. The difference in profits between net neutrality on the user side and those on the content side is \( \pi^{NU} - \pi^{NC} = \frac{c^2(1 - b_u \alpha_u^2)}{(8J)}, \)
which is positive as long as the regularity condition is satisfied and prices are nonnegative.

**Proof of Proposition 3.** Regardless of the net neutrality regime, the equilibrium profit of the ISP can be written as \( \pi(p_{c1}(c), p_{c2}(c), p_{u1}(c), p_{u2}(c), c). \) From the envelope theorem, \(-d\pi/dc = -\partial\pi/\partial c\) and, from the definition of the ISP profit, \(-\partial\pi/\partial c = q_{\alpha}\). Then, using the equilibrium prices, we find \( q_{\alpha}^N - q_{\alpha}^{NC} = c b_u \alpha_u^2/(4J) > 0, \)
\( q_{\alpha}^{NC} - q_{\alpha}^{NU} = c(1 - b_u \alpha_u^2)/(4J) > 0, \) and \( q_{\alpha}^{NU} - q_{\alpha}^D = (\alpha_c - \alpha_u)^2 b_u c/(4JL) > 0. \)

**C Extensions.**

**C.1 Mix of content**

In this extension, the demand functions become \( q_{ui} = 1 + \alpha_u[\beta q_{ci} + (1 - \beta)q_{cj}] - p_{ui} \) and \( q_{ci} = 1 + \alpha_c[\beta q_{ui} + (1 - \beta)q_{uj}] - p_{ci}, V_i \in \{1, 2\}. \) Then, \( J \) and \( L \) can be redefined as \( J = 1 - b_u \alpha_c \alpha_u (1 - 2\beta)^2 \)
and \( L = 4 - b_u (\alpha_c + \alpha_u)^2 (1 - 2\beta)^2. \)

**Proof of Lemma 2.** We maximize the ISP profit for prices, subject to the constraints of the various net neutrality regimes, and obtain the following results.

\[
\begin{align*}
W^D - W^N &= c^2(12 - (\alpha_c + \alpha_u)^2(1 - 2\beta)^2 b_u)/(4J) > 0; \\
W^{NC} - W^N &= \alpha_c^2(1 - 2\beta)^2 c^2 b_u (3 + \alpha_c(\alpha_c - 2 \alpha_u)(1 - 2\beta)^2 b_u)/(16L^2) > 0; \\
W^{NU} - W^N &= c^2(3 + \alpha_u(-2 \alpha_c + \alpha_u)(1 - 2\beta)^2 b_u)/(16L^2) > 0; \\
W^D - W^{NC} &= c^2(48 - 4(13 \alpha_c^2 + 26 \alpha_c \alpha_u + \alpha_u^2)(1 - 2\beta)^2 b_u + 8 \alpha_c(\alpha_c + \alpha_u)(\alpha_c^2 + 10 \alpha_c \alpha_u + \alpha_u^2)(1 - 2\beta)^4 b_u^2 \\
&+ \alpha_c^2(\alpha_c + \alpha_u)^2(5 \alpha_c^2 - 22 \alpha_c \alpha_u - 7 \alpha_u^2)(1 - 2\beta)^6 b_u^3 - \alpha_c^2(\alpha_c - 2 \alpha_u)(\alpha_c + \alpha_u)^4(1 - 2\beta)^8 b_u^4)]/(16J^2 L^2) > 0.
\end{align*}
\]

We also find the following.

\[
W^D - W^{NU} = (\alpha_c - \alpha_u)(c - 2\beta c)^2 b_u/(16J^2 L^2),
\]

where \( M = 20 \alpha_c - 4 \alpha_u - (3 \alpha_c^3 + 23 \alpha_c^2 \alpha_u + \alpha_c \alpha_u^2 + 5 \alpha_u^3)(1 - 2\beta)^2 b_u + \alpha_u (\alpha_c + \alpha_u)^2(2 \alpha_c^2 + \alpha_c \alpha_u + \alpha_u^2)(1 - 2\beta)^4 b_u^2 \). This is negative if and only if \( \alpha_u \in [\alpha_c, \alpha_c(b)], \) where \( \alpha_c(b) > 0 \) is such that \( M = 0. \) Finally, we have the following,

\[
W^{NU} - W^{NC} = c^2[3 - 3 \alpha_c^2(1 - 2\beta)^2 b_u - 2 \alpha_c \alpha_u (1 - 2\beta)^2 b_u \\
+ \alpha_u^2(1 - 2\beta)^2 b_u - \alpha_c^4(1 - 2\beta)^4 b_u^2 + 2 \alpha_c^3 \alpha_u (1 - 2\beta)^4 b_u^2]/(16J^2),
\]

(21)
which is positive unless $\beta$ is close to 1 and $\alpha_c$ is significantly higher than $\alpha_u$.

C.2 Affiliated content

In this extension, the profit of the ISP is given as follows:

$$
\pi = p_u q_{u1} + p_u q_{u2} + p_c q_{c1} + p_c q_{c2} + \gamma \int_0^{v_{u2}(q_{c2}(p_c,p_u))} \Phi_u(v) dv,
$$

(22)

where $\gamma$ is the share it owns in CP$_2$. We redefine $J$ and $L$ as $J = 1 - b_u \alpha_c \alpha_u$ and $L(\gamma) = 4 - \gamma - b_u (\alpha_c + \alpha_u)^2$. We also define $R = 4J L(\gamma) - \gamma L(0)$, $S = 4J L(0) - \gamma [4 - b_u \alpha_c (4 (\alpha_u - \alpha_c) + b_u \alpha_c (\alpha_c + \alpha_u)^2)]$, and $T = 8 + 4 b_u \alpha_u - 4b_u^2 \alpha_c \alpha_u + 4b_u \alpha_c (1 - 2 \alpha_u - b_u \alpha_u^2)$. We then obtain the following equilibrium quantities:

$$
q^D_{c1} = (2 + \alpha_c + \alpha_u)/L(0); \quad q^D_{c2} = (2 + \alpha_c + \alpha_u)/(L - \gamma);
$$

$$
q^N_{c1} = q^N_{c2} = (2 + \alpha_c + \alpha_u)/L(\gamma) ; \quad q^N_{u1} = q^N_{u2} = [2 - \gamma/2 + (\alpha_c + \alpha_u) b_u]/L(\gamma);
$$

$$
q^{NU}_{c1} = 2(2 + \alpha_c + \alpha_u)/(2J - \gamma)/R; \quad q^{NU}_{c2} = 4(2 + \alpha_c + \alpha_u) J/R;
$$

$$
q^{NU}_{u1} = |T - \gamma(4 + b_u \alpha_c(1 - \alpha_u) + 3b_u \alpha_u + b_u \alpha_u^2)/R; \quad q^{NU}_{u2} = |T - \gamma(4 + b_u \alpha_c(1 - 3\alpha_u) - b_u \alpha_u - b_u \alpha_u^2)/R;
$$

$$
q^{NC}_{c1} = 2(2 + \alpha_c + \alpha_u)/(2J - \gamma b_u \alpha_c^2)/S; \quad q^{NC}_{c2} = 4(2 + \alpha_c + \alpha_u) J/S;
$$

$$
q^{NC}_{u1} = |T - \gamma(2 + b_u \alpha_c(2 - \alpha_u) + b_u^2 \alpha_c^3 + b_u \alpha_c^2 (3 + b_u \alpha_u))/S; \quad q^{NC}_{u2} = |T - \gamma(2 - b_u \alpha_c(2 + 3 \alpha_u) + b_u^2 \alpha_c^3 + b_u \alpha_c^2 (1 + b_u \alpha_u))/S.
$$

(23)

We use these equilibrium quantities to conduct numerical simulations. We thereby obtain the results shown in Figure 2c and find that, for all $c, \gamma \in [0,1]$, the results of Proposition 2 remain valid.

C.3 Popular content

The existence of popular content is modeled as an upward shift in the demand for CP$_2$. We suppose, in particular, that $\Phi_{c1} = 1 + b_c (n_c q_{u1} - p_{c1})$ and $\Phi_{c2} = \delta + b_c (n_c q_{u2} - p_{c2})$, where $\delta \geq 1$. We return to the initial definitions of $J$ and $L$, that is, $J = 1 - b_u \alpha_c \alpha_u$ and $L = 4 - b_u (\alpha_c + \alpha_u)^2$, which are positive from the second-order and regularity conditions, respectively. Given the regularity condition, all prices are nonnegative if and only if $1 + J + \alpha_c - \alpha_u - b_u \alpha_u^2 > 0$, $2 + \alpha_c - \alpha_u + b_u (\alpha_c + \alpha_u)[(\delta - 1) \alpha_c - (3 + a) \alpha_u]/4 > 0$, and $8 - 2b_u (\alpha_c (1 + \delta + 2 \alpha_c) + (\delta + 2 \alpha_c - 3) \alpha_u) - b_u^2 (\delta - 1) \alpha_u (\alpha_c + \alpha_u)^2 > 0$.

Proof of Proposition 4. First, we find the following.
\[ W^N - W^D = [4 + b_u(-7\alpha_c^2 + 10\alpha_c\alpha_u + 9\alpha_u^2 + b_u(\alpha_c^4 + 2\alpha_c^3\alpha_u - 18\alpha_c^2\alpha_u^2 - 14\alpha_c\alpha_u^3 - 7\alpha_u^4)) \\
+ b_u^2\alpha_u^2(\alpha_c + \alpha_u)(2\alpha_c^2 + 2\alpha_c\alpha_u + \alpha_u^2)](\delta - 1)^2/(2JL)^2; \]

\[ W^N - W^{NC} = b_u\alpha_u[4\alpha_c + \alpha_u - b_u\alpha_c\alpha_u(\alpha_c - 2\alpha_u)](\delta - 1)^2/(4J)^2; \]

\[ W^{NC} - W^{NU} = (1 - b_u(\alpha_c - 2\alpha_u)\alpha_u)(\delta - 1)^2/(16J), \]

all of which are positive from the regularity and the nonnegativity conditions. Figure 2d shows that \( W^D \leq W^{NU} \leq W^{NC} \), \( W^{NU} \leq W^D \leq W^{NC} \), and \( W^{NU} \leq W^{NC} \leq W^D \) are all possible. This completes the proof of Proposition 4.
Table 1: Net neutrality vs. (full) discrimination based on costs, affiliated content, or popular content

<table>
<thead>
<tr>
<th>Source of discrimination</th>
<th>Total reallocation effect within sides</th>
<th>Matching effect between sides</th>
<th>Higher welfare</th>
<th>Possibly better regulation</th>
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<tr>
<td>costs</td>
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<td>positive</td>
<td>discrimination</td>
<td>user side neutrality</td>
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<tr>
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<td>negative</td>
<td>positive</td>
<td>discrimination</td>
<td>user side neutrality</td>
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<tr>
<td>popular content</td>
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<td>negative</td>
<td>net neutrality</td>
<td>none</td>
</tr>
</tbody>
</table>