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# A Role for Cheap Talk in Disclosure

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# A Role for Cheap Talk in Disclosure*

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**Abstract**

This paper studies a one-sender-one-receiver disclosure game with general receiver preferences and message structures. Drawing on techniques from information design, I provide a characterization of the Perfect Bayesian Equilibrium outcomes. I find that any PBE can be interpreted as a combination of cheap talk equilibria in a partitional form. I revisit Milgrom (1981, 2008) and identify conditions for the classic unraveling result. I provide an algorithm to construct a PBE in games with linear disclosure structure. In addition, I apply the theory to examples of labor markets and political campaigns. The theory explains why communication usually involves presentation of evidence and randomization over messages.

**JEL Classification Codes:** C72, D82, D83

*Key words:* Communication, Incomplete information, disclosure, cheap talk, Information design

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# 1 Introduction

Hard evidence is an important feature of how individuals and organizations communicate. Agents can convey valuable information by providing documents, degrees, receipts, footage, etc. For instance, a potential worker will only be able to provide a professional certificate if he has been well trained and passed a series of examinations. A researcher can only be able to provide particular statistical results from data analyses, if certain hypotheses are true. The credibility of hard evidence originates from the fact that the evidence can only be provided in certain circumstances. In light of this, when an agent receives hard evidence, she will update her belief about whether an event is true or false.

In an influential series of papers, Grossman (1981) and Milgrom (1981) introduce disclosure games: A privately informed sender (he) chooses which hard information to reveal to a receiver (she). After observing the information, the receiver chooses an action that impacts both her and the sender's payoffs. The literature on information disclosure has focused on *what evidence to reveal.* In Grossman (1981) and Milgrom (1981), this is manifested as a sender perfectly revealing his information by providing precise evidence. Verrecchia (1983), Dye (1985), Eso and Galambos (2013), and Hotz and Xiao (2013) show that the sender can also selectively present evidence — perfectly revealing the goods news while hiding the bad news. In contrast, this paper focuses on *how to present evidence.* There are two important ways in which the sender may choose to present evidence. First, the sender may randomize which evidence he sends. Second, the sender may augment the evidence with cheap talk messages. For instance, a journalist can stochastically report evidence from the set of material he collected. He may also augment that evidence by making use of natural language.

This paper studies a general disclosure game, where the sender can both randomize evidence and augment evidence with cheap talk messages. The message structure satisfies a natural condition called *strong normality.* Intuitively, it is the idea that the sender can provide any combination of the evidence he has. Strong normality encompasses almost all the message structures found in applications. The paper follows the literature in assuming that the sender's payoffs are state independent. But unlike the literature, it allows for general state-dependent receiver payoffs. (The literature typically assumes that the receiver's payoffs take specific functional forms or satisfy properties like single-peakedness or concavity.)

The main result of the paper characterizes the Perfect Bayesian Equilibrium

(PBE) outcomes of disclosure games. In equilibrium, the sender presents evidence in a way that effectively partitions the state space. So, at every state in a partition member, the sender will reveal information that is consistent with the partition member. Moreover, often, the sender may provide evidence that is inconsistent with other partition members. (When other partition members are consistent with the evidence, an incentive compatibility condition will ensure that the sender does not mimic the evidence in other states.) As a consequence of this partitioning, the disclosure game can be divided into auxiliary games, with each partition member acting as a state space. Within each auxiliary game, the sender randomizes evidence and cheap talk messages. This leads the receiver to optimize in a way that makes the sender indifferent at every state in the same partition member.

This equilibrium characterization is familiar from the cheap-talk literature. Lipnowski and Ravid (2020) study cheap talk with state-independent sender preferences. They show that the equilibrium is characterized by the sender randomizing over messages and getting paid equally. By contrast, this paper studies an information disclosure game. It shows that the sender will use hard evidence to partition the state space and, given the partition member, the equilibrium behavior will coincide with the cheap talk equilibrium.[1] Put differently, a PBE outcome of the disclosure game is equivalent to a combination of cheap talk equilibrium outcomes in separated auxiliary games.

To characterize the PBE outcomes, the paper borrows techniques from the information design literature. By doing so, I can geometrically characterize the *ex ante* distributions of the receiver's beliefs and the sender's payoffs. Unlike the information design literature, here, the sender has incentive constraints at each state.[2] Because the sender's payoffs are state-independent, these incentive constraints can be translated into feasibility constraints on the geometric formulation of equilibrium outcomes. Section 5 discusses this method in detail.

Using the method developed here, the paper studies disclosure under two specific assumptions about the message structure: *full verifiability* and *linear disclosure*. Under full verifiability, the sender can prove *any* true event. The paper shows that full disclosure is the unique equilibrium outcome, when the message structure satisfies full verifiability, the sender's preferences are monotonic, and the receiver's preferences satisfy diminishing marginal utility. This significantly generalizes the

[1]The partitional equilibrium in Crawford and Sobel (1982) originates from conflicts of interest between the sender and receiver. In this paper, the partition is due to the property of hard evidence.

[2]This is not the case in the information design literature, where the sender commits to a signaling device.

classic unravelling result in Milgrom (1981, 2008), which assume that the receiver's payoffs satisfy a single-crossing condition. Seidmann and Winter (1997), Mathis (2008), and Giovannoni and Seidmann (2007) generalize the unraveling result in a different direction. They extend the result to the case of state-dependent sender preferences and require stronger conditions than this paper. Hagenbach, Koessler, and Perez-Richet (2014) explore sufficient conditions for the existence of a fully revealing equilibrium, but not necessarily a unique fully revealing equilibrium.

In linear disclosure, the states are linearly ordered and superior states have more messages. For instance, suppose the states reflect personal wealth $w$ and the sender can verify that his wealth is at least $v \leq w$. Then higher levels of wealth are associated with more abundant a set of available messages. In this case, the equilibrium outcome reduces to partition members that are increasing both in the disclosure order and in sender payoffs. Drawing on the securability result in Lipnowski and Ravid (2020), I provide an algorithm that generates an equilibrium in this class of games.

The remainder of this paper is organized as follows. Section 2 summarizes the related papers in the literature. Section 3 gives a numerical example to provide intuition for the general results. Section 4 lays out the basic model. Section 5 presents the main result, i.e., the characterization of PBE outcomes. Section 6 focuses on the case of full verifiability and generalizes the "unraveling" result. Section 7 focuses on the case of linear disclosure and develops an algorithm to generate an equilibrium. Section 8 applies the theory to an example of political campaigns. Section 9 concludes.

# 2 Literature Review

The extant literature on disclosure has been focusing on how to maximize information revelation and the receiver's welfare. The pioneering papers, Milgrom (1981) and Grossman (1981), demonstrate that when a seller wants to convince a buyer that his product has high quality, he has no choice but disclosing all information. Otherwise, a rational buyer would assume the worst and take an action unfavorable to the sender. The following papers explore conditions for sustaining equilibria with full information in a class of games that relax the assumption of state-independent sender payoff (Seidmann and Winter, 1997; Mathis, 2008; Giovannoni and Seidmann, 2007). Another branch of the literature lies at the intersection of disclosure and mechanism design. Green and Laffont (1986) and Bull and Watson (2007) prove the extended revelation principle that any implementable social choice function can be implemented by a direct mechanism under which the sender provides the most precise evidence. In addition, there are recent papers on disclosure that focus on finding conditions under which the optimal mechanism for the receiver does not bring her a payoff higher than the maximal equilibrium payoff (Hart, Kremer, and Perry, 2017; Ben-Porath, Dekel, and Lipman, 2019).

In contrast to the receiver, the analysis of the sender's behavior is relatively simple. In most papers mentioned in the last paragraph, it is without loss to think of the sender providing as much evidence as he can. Even in the studies where information is not fully revealed (Verrecchia, 1983; Dye, 1985; Eso and Galambos, 2013; Hotz and Xiao, 2013; Rappoport, 2020), the sender discloses the true state in a certain range of states but pools the rest of states. This paper extends the analysis of the sender's disclosure behavior to the study of how he may randomize evidence. Sometimes, as in Section 3, a mixed strategy of the sender is prescribed by the unique equilibrium.

One related paper is Forges and Koessler (2008), which provide a geometric characterization of the equilibrium outcomes in a repeated disclosure game under the assumption of arbitrary state-dependent sender preferences. Different from the current paper, they study the extensive-form of disclosure games and restrict attention to the message structure with full verifiability.

There are recent communication models (Kamenica and Gentzkow, 2011; Lipnowski and Ravid, 2020) that characterize the geometric formulation of equilibrium outcomes. Kamenica and Gentzkow (2011) study a persuasion model in which the sender does not have private information and he can choose from any signaling devices to send signals. They demonstrate that the maximal payoff to a sender is on the concave closure of his value function on the belief space. The difference in this paper is that I assume private information to the sender, so that the sender's strategy has to satisfy incentive constraint at each state. Lipnowski and Ravid (2020) apply the geometric method to cheap talk with state-independent sender payoff. They find that an equilibrium takes a simple form: the sender randomizes over messages and receives the same payoff from sending each message. This is exactly what happens in each partitional member in a disclosure game. Besides, Lipnowski and Ravid (2020) give a result to identify cheap talk equilibria, which will be used in Section 7 to provide an algorithm to generate a PBE in a linear disclosure game.

# 3 Example

This section presents three examples to illustrate general features of equilibrium of a disclosure game. The first example shows that any equilibrium can be interpreted as a combination of cheap talk equilibria in a partitional form. The second example stresses the lower bounds for the sender's equilibrium payoffs in each partition member based on the message structure. The third example focuses on the importance of the linking between the sender's payoffs across partition members, so that the sender does not want to mimic states outside the partition member his state belongs to. As will be shown in Section 5, these examples correspond to three conditions in the characterization of PBE outcomes.

## 3.1 Labor Market I

A firm is recruiting a worker for a position in either the Finance Department (F) or the Accounting Department (A). In addition, the firm has the option to reject the worker's application (R). The worker prefers finance, to accounting, to being rejected. The firm's preference depends on the quality of the worker: the worker can either be unskilled (u), a skilled financial analyst (f) or a skilled accountant (a).[3] Suppose a skilled employee is productive only in his profession. So that a financial analyst will bring the firm a net benefit of 1 in F and an accountant 1 in A. If the worker is mismatched or unskilled, he will not generate any return; instead, he will incur some costs to the firm for wasting resources in training and for slowdown in teamwork.

Table 1 presents the payoffs for the worker and the firm. The first argument of each table entry is the payoff to the worker, and the second is that to the firm. Let $\mu = (\mu_u, \mu_f, \mu_a)$ be the probabilities of the applicant's being unskilled, a financial analyst, and an accountant. The state is private information for the worker, and there is a common prior $\mu_0 = (\frac{1}{2}, \frac{1}{4}, \frac{1}{4})$.

Suppose for a financial analyst or an accountant, he has a degree $d$ that he can present to ensure his skillfulness. (Yet $d$ does not indicate which skill he has.[4])

[3] Including another type which is good at both finance and accounting will not change the insight of this example, when the probability of this type is low. But adding it will significantly complicate the analysis, so for simplification I choose not to include this type.

[4] If d differentiates between the finance and accounting types, full information is one equilibrium outcome. Here I focus on a case where the evidence is coarse. In the next two examples, Labor Market II and Labor Market III, I will further change the message structure and examine its effect on the equilibrium outcomes. The theory, however, allows me to study all variants on the current message structure satisfying the conditions specified in Section 4.

| | | Firm's Action | | |
|---|---|---|---|---|
| | | $F$ | $A$ | $R$ |
| | $f$ | 2,1 | 1,-2 | 0,0 |
| State | $a$ | 2,-2 | 1, 1 | 0,0 |
| | $u$ | 2,-2 | 1,-2 | 0,0 |

Table 1: Payoffs for the Worker and the Firm

Furthermore, the worker can always send two cheap talk messages, $m_F$, meaning "I want to go to the Finance Department," and $m_A$, meaning "I want to go to the Accounting Department." The worker can simultaneously present $d$ and send a message $m_F$ or $m_A$. For simplicity of exposition, I denote by $d_F$ (resp. $d_A$) the combination of $d$ and $m_F$ (resp. $m_A$). In summary, a skilled type — a financial analyst or an accountant — has a message set $\{m_F, m_A, d_F, d_A\}$, while the unskilled type has only $\{m_F, m_A\}$.

By default the firm will reject the application, so a burden is on the worker to reveal information about his types. But he is in a situation where either sending messages or presenting the degree alone does not change the firm's action. If he only sends messages, it reduces to cheap talk — the information is not credible enough.[5] If he only presents evidence when he is a skilled type, the firm knows he is skilled but is unsure of which skill he has, so it still cannot make him an offer.

Below I will present the Pareto optimal equilibrium of this game. Consider the following strategy profile: in $f$, the worker presents $d_F$, and in $a$, the worker randomizes between $d_A$ and $d_F$ with probability half-half. Upon receiving $d_F$, the firm updates its belief to $(0, \frac{2}{3}, \frac{1}{3})$. With this belief it is indifferent between $F$ and $R$. Let the firm randomize between $F$ and $R$ with probability half-half; it leads to expected payoff 1 for the worker. After $d_A$, the firm is sure that the worker is good at accounting, so it makes an offer $A$. The payoff to the worker is also 1. This is a PBE, because in $f$ and $a$, the expected payoffs to the worker from sending both $d_F$ and $d_A$ are equal to 1, which justifies the randomization in $a$. In $u$, the worker has no access to $d_F$ and $d_A$, and therefore cannot mimic $f$ or $a$. This is the Pareto-optimal equilibrium outcome because the skilled types have reached their maximum payoff and the firm has seen the maximal amount of information being revealed.[6]

[5]That is because that the prior probability of $u$ is high. When $\mu_0(u) \leq \frac{1}{3}$, the sender's preferred equilibrium is that the unskilled type can partially pool with the finance type and partially pool with the accounting type. One can find this "cheap talk" equilibrium by the method developed in Lipnowski and Ravid (2020).

[6]As will be discussed later, the skilled types reach the highest payoff that is described by the quasi-concave closure of the worker's continuation payoff. The firm has maximal information in

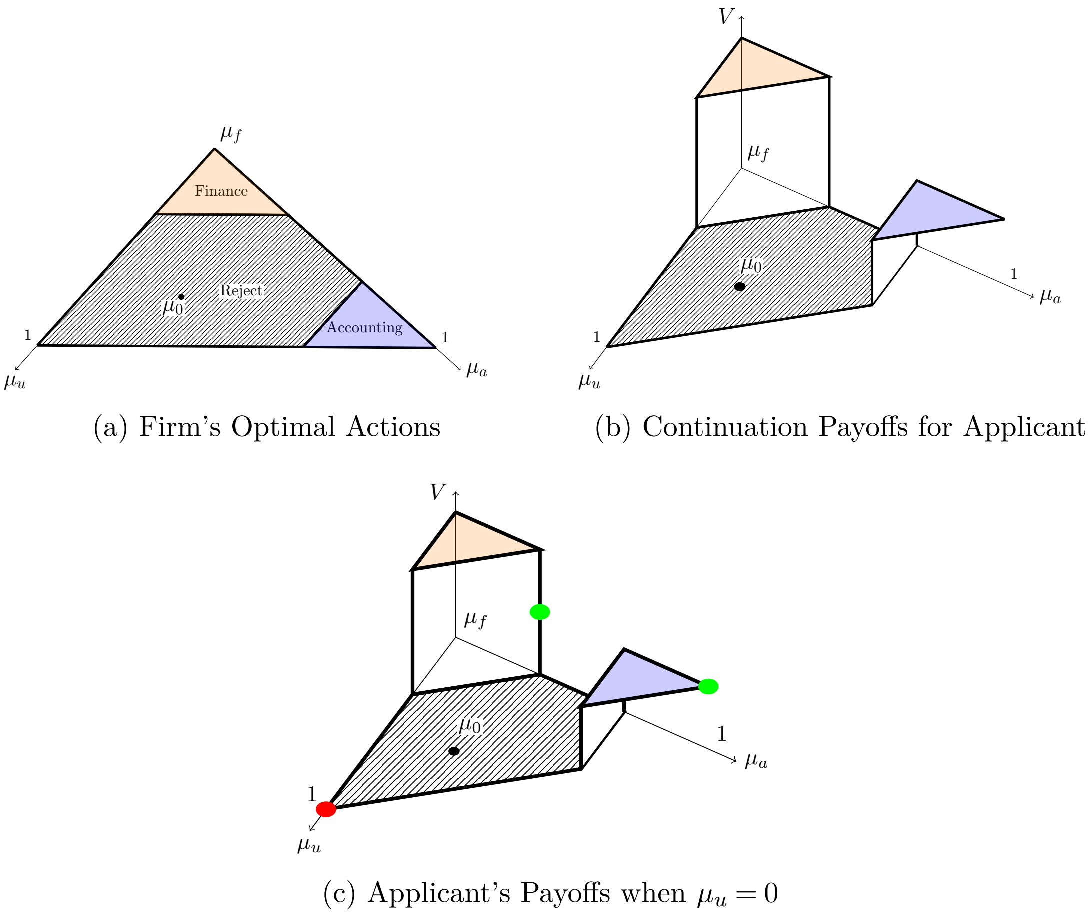



Figure 1: The Pareto Optimal Equilibrium of the Labor Market

Next, I will introduce the geometric approach to reach this conclusion that will also be applicable to a broad class of games. By taking this approach, I can use convenient geometric properties of an equilibrium to solve for the outcomes. To understand the approach, note the firm's decision depends on its assessment of the worker's quality. Figure 1(a) shows the best response of the firm to each belief. Figure 1(b) shows the continuation payoffs to the worker given the firm's best responses. The colored areas correspond to the worker's payoffs, given the firm's different best responses. For instance, when the firm's belief lies in the orange area, it will offer F, and the payoff for the worker is 2. On the borders between the areas, the firm randomizes between actions, resulting in a continuum of possible payoffs for the worker.

Again focus on the Pareto-optimal equilibrium where a skilled type wishes to

Blackwell order. Wu (2018) proves that an information structure is more informative when the distribution of its induced posterior beliefs is more "dispersed" under certain conditions. This strategy induces a distribution of posterior beliefs that spread out more than any other equilibria.

separate from a non-skilled type by providing evidence. That means in equilibrium, the employer is sure about whether the worker is skilled or not. If the firm learns the worker is skilled, it will assign probabilities half-half to $f$ and $a$. The conditional probability $(\mu_f, \mu_a) = (\frac{1}{2}, \frac{1}{2})$ is illustrated by the black dot in Figure 1(c). Now I can regard $\{f, a\}$ as the state space of an auxiliary game with a prior $(\frac{1}{2}, \frac{1}{2})$. The previous strategy profile induces randomization over belief points as illustrated by the green points in Figure 1(c). The domain of the blue points are posterior beliefs of the firm, and the expected payoff to the worker at each belief point is 1.

The equilibrium outcome restricted in $\{f, a\}$ coincides with a cheap talk equilibrium in Lipnowski and Ravid (2020), that characterize cheap talk equilibria under sender state-independent preferences. They show that every equilibrium outcome takes the form of a feasible distribution over posterior beliefs with equal associated payoffs for the sender. Here this condition is satisfied, provided that, in Figure 1(c), the payoff associated with each blue point is 1. The difference in disclosure is that the equilibrium takes a partitional form. One auxiliary game has a trivial state space $\{u\}$, and another a state space $\{f, a\}$. The equilibrium outcome within each game is equivalent to a cheap talk equilibrium. As will be shown in Section 5, this is a general feature of any equilibrium of disclosure games.

In light of this structure, I can translate the question of finding the worker's favorite equilibrium into that of finding his favorite cheap talk equilibrium in each partition member. According to Lipnowski and Ravid (2020), the maximum (expected) payoff a sender can achieve with cheap talk messages can be expressed by the quasi-concave closure of his value function. In this example, the quasi-concave closure conditional on the fact that the worker is skilled is represented by the red curve in Table 1(c). Therefore, given the conditional probability, the skilled types' maximum payoff is 1 and the equilibrium described above is the most preferred by the worker.

Furthermore, this example also shows how equilibria differ in related communication games. In Bayesian persuasion, the worker has commitment power and there is an equilibrium where the unskilled can partially pool with the financial analyst and partially with the accountant. The equilibrium posteriors are $(1, 0, 0)$, $(\frac{1}{3}, \frac{2}{3}, 0)$, and $(\frac{1}{3}, 0, \frac{2}{3})$. If it is a cheap talk, then communication is impossible, no information is transmitted, and the payoff is 0 for both the worker and the firm.

| | | Firm's Action | | |
|---|---|---|---|---|
| | | $F$ | $A$ | $R$ |
| | $f$ | 2,2 | 1,0 | 0,0 |
| State | $a$ | 2,0 | 1,1 | 0,0 |
| | $u$ | 2,-2 | 1,-2 | 0,0 |

Table 2: Payoffs for the Worker and the Firm

## 3.2 Labor Market II

In the above example, for any $s \in [0, 1]$, there is an equilibrium where the worker's payoff is $s$ at $f$ and $a$. The equilibrium strategy is as follows. In $f$, the worker sends $d_F$ and $d_A$ with probabilities $\frac{2}{3}$ and $\frac{1}{3}$, and in $a$, the worker sends $d_F$ and $d_A$ with probabilities $\frac{1}{3}$ and $\frac{2}{3}$. Upon receiving $d_F$, the firm plays $F$ with probability $\frac{s}{2}$ and $R$ with the complementary probability; upon receiving $d_A$, the firm plays $A$ with probability $\frac{s}{1}$ and $R$ with the complementary probability. In $u$, the worker can only send cheap talk messages, which is followed by rejection from the firm.

But the equilibrium outcomes are sensitive to change in the message structure. Suppose that the accounting type has another certificate $c$ that can identify his own type, everything else unchanged. Then, the sender's equilibrium payoff, either in $f$ or $a$, takes on a unique value 1. The reason is that the accounting type can guarantee himself payoff 1 by presenting $c$, which rules out other lower equilibrium payoffs.

This example suggests that there is a lower bound for the sender's equilibrium payoff in each partition member. Because evidence can effectively transmit information even *off* the equilibrium path, the sender can leverage available evidence to achieve a higher payoff than in cheap talk.

## 3.3 Labor Market III

Suppose I make changes in the labor market example in the following ways: (1) the common prior $\mu_0 = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$; (2) the preferences are presented in Table 2; (3) the available messages in $u$, $f$, and $a$ are $\{m\}$, $\{m, e_1\}$, and $\{m, e_2\}$, respectively.

Since now the finance and accounting types have evidence to distinguish themselves, full information is one equilibrium outcome. But is it an equilibrium if the unskilled type stands alone and the finance and accounting types pool together? It is plausible because this structure consists of two cheap talk equilibria: in $\{u\}$, the firm plays $R$, and in $\{f, a\}$, the firm plays $F$. Also, the expected payoff 2 is the highest payoff the worker can expect in this game.

But the worker's payoffs, if in equilibrium, are not compatible with the message structure. If $f$ and $a$ pool, the only possible message that the worker can use for this pooling is $m$ — the common message shared by $f$ and $a$. Yet once $m$ is being used, the unskilled type is able to mimic the skilled types, so that the worker should get paid no lower in $u$ than in $f$ and $a$. This contradicts that the skilled types get paid 2, while the unskilled type 0.

This example suggests the necessity for the third condition, in addition to the partitional form and lower bounds, on the linking between the sender's payoffs across partition members. In Section 5, I will summarize this condition in Lemma 3. One surprising finding is that, even though the condition is concerned with strategy and message structure, it can be directly imposed on the geometric formulation of the PBE outcomes.

# 4 Model

There is a finite state space $\Omega$. Let $\Delta(\Omega)$ be the set of probability measures on $\Omega$. An *event* $E$ is a nonempty subset of $\Omega$, and $\Delta(E)$ denotes the set of probability measures in $\Delta(\Omega)$ with support in $E$. There is a full support common prior, denoted by $\mu_0 \in \text{int}(\Delta(\Omega))$.

There are two players, a sender (S) and a receiver (R). The timing is as follows. Nature picks a state $\omega \in \Omega$ according to $\mu_0$. Then S learns $\omega$ and provide evidence to R. I model evidence by having the set of available messages depend on the state.[7] For each $\omega$, *the messages available to S at* $\omega$ are contained in a countable set $M(\omega)$. Let $M = \bigcup_{\omega\in\Omega} M(\omega)$ be the set of all messages. Upon receiving a message, R takes an action from a finite set $A$. In Sections 6 and 7, I relax the action set to be a compact metrizable space. I often write $a$ for a mixed action , i.e., $a \in \Delta(A)$.

S has a (continuous) state-independent payoff function $u^S : A \to \mathbb{R}$, while R has a (continuous) state-dependent payoff function $u^R : A \times \Omega \to \mathbb{R}$. Denote the players' *expected* payoff functions by $U^S : \Delta(A) \to \mathbb{R}$ and $U^R : \Delta(A) \times \Delta(\Omega) \to \mathbb{R}$.[8]

[7] The state-dependent message set is also assumed in language games (Blume and Board, 2013). This class of games have two differences. First, aside from the state, the message structure is also private information to players. Second, the restraint on a message set not only constrains message availability, but also players' understanding of messages.

[8] $U^R$ refers to R's *interim payoff* after she receives a message, so her (mixed) action is independent of her belief.

## 4.1 Message Structure

Say that a message $m$ *verifies* an event $E$ if (1) for each $\omega \in E$, $m \in M(\omega)$, and (2) for each $\omega \notin E$, $m \notin M(\omega)$. Or equivalently, $E = \{\omega \in \Omega \,|\, m \in M(\omega)\}$. Write $m_E$ for a generic message that verifies $E$ and $\mathscr{M}(E)$ the set of messages that verify $E$. Note that $m_E$ is consistent with each state in $E$, and inconsistent with each state not in $E$. In this sense, it provides evidence for $E$.[9] Conversely, call the event $E$ that is verified by $m_E$ the *inference* from $m_E$.

A message that verifies a state $\omega$ is denoted by $m_{\{\omega\}}$, which induces a degenerate probability distribution, $\delta(\omega)$, that puts all weight on $\omega$. I define the set of inferences of messages that are available at $\omega$ as $\mathscr{F}(\omega) = \{E \subseteq \Omega \,|\, \exists m_E \in M(\omega)\}$. Following from the definition of $\mathscr{M}(E)$, the set of available messages at $\omega$, $M(\omega)$, is composed of all evidence that can be used to verify $E \in \mathscr{F}(\omega)$.

**Lemma 1.** *For each $\omega \in \Omega$, $M(\omega) = \bigcup_{E \in \mathscr{F}(\omega)} \mathscr{M}(E)$.*

Note, for a given $E$, $\mathscr{M}(E)$ may be empty, i.e., S may not be able to verify $E$. Suppose that the true state lies in $E$ and S would like to verify $E$ but cannot. Then he might use a message $m_K$ for some $K \supsetneq E$. This message is consistent with $E$ and rules out certain irrelevant states (when $K \neq \Omega$), i.e., states in $\Omega \backslash K \subseteq \Omega \backslash E$. Absent strategic considerations, it increases the conditional probability of $E$. As such, I refer to such a message $m_K$ as *weak evidence* for $E$. Let $\mathscr{M}^*(E) = \{m_K \in M \,|\, E \subseteq K\}$ be the set of messages that provide weak evidence for $E$. Note that $m_E$ is also weak evidence for $E$.

Throughout the paper, I will impose two assumptions on message structure: *rich language* and *strong normality.*

**Definition 1.** *S has a* rich language *if (1) $|\mathscr{M}(\Omega)|$ is countably infinite, and (2) for each event $E \subsetneq \Omega$, either $\mathscr{M}(E) = \emptyset$ or $\mathscr{M}(E)$ is countably infinite.*

The first condition says that there is a countable infinity of cheap talk messages, i.e., the messages in $\mathscr{M}(\Omega)$ that are universally available. The second condition says that each piece of evidence, if it exists, has a countable infinity of copies. At first glance, the second condition may appear restrictive; but it is not. To understand why, suppose there is a single piece of evidence for $E$. Then S can combine this evidence with cheap talk messages to create equivalent copies. For example, when presenting the evidence, S can say "This is the evidence to support $E$," "Based on

[9]The meaning of "evidence" is broader than a "verifying" message. A message $m_F$, s.t. $F \subsetneq E$ is evidence for $E$, yet I do not say that it verifies $E$.

this evidence, $E$ is true," "I can prove that $E$ is true," etc. Each combination of evidence and a statement forms a distinct message with the same substance.

The second assumption is called strong normality. The idea is that if S can provide R with more than one piece of evidence, he should be able to present any *combination* of them. The inference of such a combination should be the conjunction of the inferences of different pieces of evidence.

**Definition 2.** *A message structure is* strongly normal *if for each pair of events $E, E' \subset \Omega$ with $E \cap E' \neq \emptyset$, $\mathscr{M}(E)$ and $\mathscr{M}(E') \neq \emptyset$, it is the case that $\mathscr{M}(E \cap E')$ is nonempty.*

To understand the idea, suppose that, at a state $\omega$, S can verify $E$ and $E'$, i.e., $E, E' \in \mathscr{F}(\omega)$. Then by providing the joint evidence $(m_E, m_{E'})$,[10] S can verify that (1) $(m_E, m_{E'})$ is consistent with each state in $E \cap E'$, and (2) $(m_E, m_{E'})$ is inconsistent with $\Omega \backslash (E \cap E')$. Thus, $\mathscr{M}(E \cap E')$ should be nonempty.

Strong normality is related to a widely used assumption, normality,[11] which embodies the idea that S should be able to present *all* evidence in his possession.

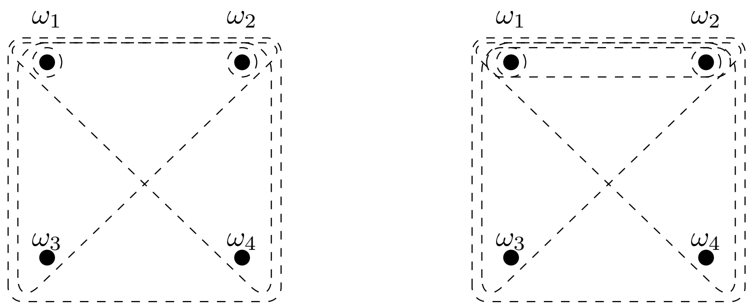
**Definition 3.** *A message structure is* normal *if for each $\omega \in \Omega$, $\mathscr{M}(\bigcap_{E \in \mathscr{F}(\omega)} E)$ is nonempty.[12] That is, the intersection of the inferences of all messages at $\omega$ can be verified.*

Figure 2 illustrates the difference between these two concepts. There, $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$. A message is identified with a set of states consistent with that message - thus its inference is represented by dashed lines. For instance, in Figure 2(a), there are five messages corresponding to the events $\{\omega_1\}$, $\{\omega_2\}$, $\{\omega_1, \omega_2, \omega_3\}$, $\{\omega_1, \omega_2, \omega_4\}$, and $\Omega$. When the state is $\omega_1$, S can verify $\{\omega_1\}$, $\{\omega_1, \omega_2, \omega_3\}$, $\{\omega_1, \omega_2, \omega_4\}$, and $\Omega$. Of course, $m_\Omega$ represents a cheap talk message. If S presents all evidence he has, he would verify $\{\omega_1\}$. The existence of evidence to verify $\{\omega_1\}$ corresponds to the fact that the message structure is consistent with normality. But it violates strong normality. Think of S simultaneously presenting evidence that verifies $\{\omega_1, \omega_2, \omega_3\}$ and $\{\omega_1, \omega_2, \omega_4\}$; then R should conclude that the state is either $\omega_1$ or $\omega_2$. Strong normality requires that S can achieve this effect through a message that verifies $\{\omega_1, \omega_2\}$. Yet under this message structure, he is unable to do so. Figure

[10]In the game, only one message is allowed to be sent, so the representation of joint evidence is only for the sake of interpretation.

[11]See Lipman and Seppi (1995), Bull and Watson (2007), Hart et al. (2017), and Ben-Porath et al. (2019).

[12]Normality is expressed in different ways in Lipman and Seppi (1995) and Bull and Watson (2007), but this definition is essentially the same as theirs.




(a) Normality (b) Strong Normality

Figure 2: Comparing Normality and Strong Normality

2(b) adds an event that verifies $\{\omega_1,\omega_2\}$; that message structure satisfies strong normality.

Because a cheap talk message, $m_\Omega$, is weak evidence for every event, $\mathscr{M}^*(E)$ is nonempty, for each $E \subseteq \Omega$. Furthermore, strong normality implies that among all weak evidence for $E$, there is one piece of weak evidence, namely maximal weak evidence, that conveys the most precise information.

**Definition 4.** *A message $m_F \in \mathscr{M}(F)$ is* maximal weak evidence *for $E$ if:*

1. *$E \subseteq F$.*

2. *For each $m_{F'} \in \mathscr{M}(F')$ s.t. $E \subseteq F'$, $F \subseteq F'$.*

*I use $m^*(E)$ to denote maximal weak evidence for $E$.*

Let $\mathscr{W}(E) = \{K \subseteq \Omega \,|\, \exists m_K \in \mathscr{M}^*(E)\} = \{K \supseteq E \,|\, \mathscr{M}(K) \neq \emptyset\}$ include the inferences of all weak evidence for $E$. If S presents $m^*(E)$, it is equivalent that he provides all the weak evidence for $E$ in the meantime. Therefore, $m^*(E)$ verifies $\bigcap_{K\in\mathscr{W}(E)} K$. Write *the inference of* $m^*(E)$ as $E^* = \bigcap_{K\in\mathscr{W}(E)} K$.

**Lemma 2.** *For each $E \subseteq \Omega$, there is maximal weak evidence for $E$. Moreover, $m^*(E)$ verifies $E^*$.*

For each $\omega \in E^*$, I call $\omega$ an *indistinguishable* state from $E$. That means whenever S provides weak evidence for $E$, the information does not rule out the possibility of $\omega$. Alternatively, it means that S has access to *all* weak evidence for $E$ at $\omega$.

Take the message structure in Figure 2(b) for example. There is no message that verifies $\{\omega_2,\omega_4\}$, but there is weak evidence for the event that verifies $\{\omega_1,\omega_2,\omega_4\}$.

It is also maximal weak evidence for $\{\omega_2,\omega_4\}$, i.e., $\{\omega_2,\omega_4\}^* = \{\omega_1,\omega_2,\omega_4\}$. So the indistinguishable states from $\{\omega_2,\omega_4\}$ are $\omega_1,\omega_2$, and $\omega_4$.

## 4.2 Equilibrium

This paper focuses on the solution concept of Perfect Bayesian Equilibrium (PBE). A strategy of S is a mapping $\sigma : \Omega \to \Delta(M)$, a strategy of R is a mapping $\rho : M \to \Delta(A)$, and a belief system is a mapping $\phi : M \to \Delta(\Omega)$. A PBE $\mathcal{E}$ is a triple $(\sigma,\rho,\phi)$ satisfying following conditions.

**Definition 5.** *A profile* $(\sigma,\rho,\phi)$ *is a* Perfect Bayesian Equilibrium *if the following hold:*

1. *For each* $m \in M$, $\rho(m) \in \arg\max_{a\in\Delta(A)} U^R(a,\phi(m))$.
2. *For each* $\omega \in \Omega$ *and* $m \in \operatorname{supp}(\sigma(\omega))$, $m \in \arg\max_{m'\in M(\omega)} U^S(\rho(m'))$.
3. *For each* $m \in \bigcup_{\omega\in\Omega} \operatorname{supp}(\sigma(\omega))$, $\phi(m)$ *is updated by Bayes rule.*

A PBE $(\sigma,\rho,\phi)$ is called a *fully revealing equilibrium* if, for each $\omega \in \Omega$ and $m \in \operatorname{supp}(\sigma(\omega))$, $\phi(m)(\omega) = 1$. In a fully revealing equilibrium, R is always able to correctly infer the true state. On the contrary, if in a PBE, for each $\omega \in \Omega$ and $m \in \operatorname{supp}(\sigma(\omega))$, $\phi(m)(\omega) = \mu_0$, then I say that the PBE is *uninformative.* If a PBE is neither fully revealing nor uninformative, I call it a *partially revealing equilibrium.*

# 5 Characterization of PBE outcomes

In this section I use techniques from *information design* to characterize the PBE outcomes. Instead of directly characterizing PBE in Definition 5, I propose an equivalent formulation of PBE that characterizes distribution of posterior beliefs and associated payoffs. Unlike the information design literature - where S can choose freely from arbitrary information structures - here his choice is restricted by feasibility constraints. The key is that these constraints can be incorporated into the geometric approach.

Define the *best response correspondence* of R as $r : \Delta(\Omega) \rightrightarrows \Delta(A)$. So, $r(\mu)$ gives the set of optimal (mixed) actions to the belief $\mu \in \Delta(\Omega)$, i.e., $r(\mu) = \arg\max_{a\in\Delta(A)} U^R(a,\mu)$. Given the optimal response from R to $\mu$, the set of S's possible payoffs is $V(\mu) = \{U^S(a) \,|\, a \in r(\mu)\}$. Call $V : \Delta(\Omega) \rightrightarrows \mathbb{R}$ the *value correspondence* for $S$. By Berge's

Maximum Theorem, for each $\mu \in \Delta(\Omega)$, $V(\mu)$ is nonempty, compact, and convex, and $V$ is upper hemicontinuous.

To capture the informational content of the sender's strategy, define an *information policy*, $\tau \in \Delta(\Delta(\Omega))$, as an *ex ante* distribution of beliefs, whose expectation is equal to the prior, i.e., $\mathbb{E}[\tau] = \mu_0$. Let

$$\mathcal{I} = \{\tau \in \Delta(\Delta(\Omega)) \,|\, \mathbb{E}[\tau] = \mu_0 \textit{ and } |\text{supp}(\tau)| \textit{ is countable}\}$$

denote the set of all information policies.[13] Also, I define a *conditional information policy* in $E$, $\tau_E \in \Delta(\Delta(E))$, to be an *ex ante* distribution of beliefs such that $\mathbb{E}[\tau_E] = \mu_0(\cdot|E)$.

Any strategy $\sigma$ of S induces such an information policy. Conversely, any information policy can be generated by some strategy $\sigma$ (Aumann and Maschler, 1995; Kamenica and Gentzkow, 2011).[14] Instead of studying S's strategy, I will explore the features of the corresponding information policy.

Next, I introduce notation for the payoffs to S conditional on the realization of posterior beliefs. Let $\lambda : \Delta(\Omega) \to \mathbb{R}$ be the equilibrium payoff function that associates each equilibrium belief with a continuation payoff for S. But it does not incorporate the expected payoff for $R$. Call $(\tau, \lambda)$ a *PBE outcome* if it is induced by a PBE $(\sigma, \rho, \phi)$. To be concrete, that means $\tau$ is induced by $\sigma$, and for each $\mu \in \text{supp}(\tau)$ that is induced by $m \in M$, $\lambda(\mu) = U^S(\rho(m))$.

Then I discuss the necessary conditions to support $(\tau, \lambda)$ as a PBE outcome. First note that for each $\mu \in \text{supp}(\tau)$, $\lambda(\mu) \in V(\mu)$, which captures that R best responds to her belief. With this in mind, I focus on the constraints on S.

Before I dive into the details, I make an important observation that any PBE takes a partitional form. Since S's payoff only depends on the receiver's action, in any two states if S uses a message in common, he gets paid equally in these two states. This equivalence relation induces equivalence classes among the state space, which form a partition of $\Omega$. Conversely, any message comes from one equivalence class, so that in equilibrium, the equivalence classes coincide with informational partition.

I denote a partition of the state space $\Omega$ by $\mathscr{P}$. If an information policy $\tau$ can be split into conditional information policies in each partition element of $\mathscr{P}$, $\{\tau_P\}_{P \in \mathscr{P}}$, I say that $\tau$ is *governed* by partition $\mathscr{P}$. Formally, it means that for

[13] Since there are countable messages in this game, information policies have countable supports.

[14] The formula that gives how to find a strategy $\sigma$ that induces a certain information policy $\tau$ is provided in Kamenica and Gentzkow, 2011, pg.2596.

each $\mu \in \text{supp}(\tau)$, $\text{supp}(\mu) \subseteq P$ for some $P \in \mathscr{P}$. In equilibrium, $\tau$ is governed by a partition $\mathscr{P}$, so that for each $P \in \mathscr{P}$, and each $\mu, \mu' \in \text{supp}(\tau_P)$, $\lambda(\mu) = \lambda(\mu')$. To simplify exposition, I use $\lambda_P$ to represent *the equilibrium payoff to S in a partition member $P$*.

Now I discuss the condition that S does not want to mimic other states. Under a partition $\mathscr{P}$, suppose $\omega \in \Omega$ is contained in $P \in \mathscr{P}$. In $\omega$, though S has no incentive to mimic any $\omega' \in P$ due to indifferent payoffs, he may want to mimic some $\omega'' \in P' \neq P$. This calls for a payoff relation between $\{\lambda_P\}_{P \in \mathscr{P}}$ to support the equilibrium.

If in $\omega \in P$, S can send a message that is being used in $\omega' \in P'$, that means $\lambda_P \geq \lambda_{P'}$. But this discussion depends on the message structure and the strategy of S. It is not clear, however, what condition to put on a pair $(\tau, \lambda)$. The challenge is that there are many ways to induce the same information policy by using different sets of messages. So it is hard to relate the realization of a belief to the message availability in other states. Fortunately, I only need to focus on the maximal weak evidence for the information, regardless of how complicated the message structure is.

Suppose $(\tau, \lambda)$ is a PBE outcome and $\tau$ is governed by $\mathscr{P}$. For a given $\mu \in \text{supp}(\tau)$, $\text{supp}(\mu) \subseteq P \in \mathscr{P}$. Suppose $\mu$ is induced by a message $m_E$, that means S must send $m_E$ at each $\omega \in \text{supp}(\mu)$, as prescribed by his strategy $\sigma$. It implies that $m_E \in M(\omega)$, for each $\omega \in \text{supp}(\mu)$. So by the definition of verifiability, $E \supseteq \text{supp}(\mu)$ and $m_E$ is weak evidence for $\text{supp}(\mu)$. By Lemma 2, there is maximal evidence $m^*(\text{supp}(\mu))$ for $\text{supp}(\mu)$. Since $m^*(\text{supp}(\mu))$ is more precise than $m_E$, it has a narrower inference, i.e., $\text{supp}(\mu)^* \subseteq E$. Then, for each state $\omega \in \text{supp}(\mu)^*$, I have $\omega \in E$. Thus, $m_E \in M(\omega)$. As a result, S has access to the message used in $P$; if $\omega \in P'$, $\lambda_{P'} \geq \lambda_P$. Notice that this condition holds whenever $\mu$ is realized in equilibrium, irrespective of which message is being used to induce $\mu$.

Therefore, I have the following result.

**Lemma 3.** *Suppose $(\tau, \lambda)$ is a PBE outcome and $\tau$ is governed by $\mathscr{P}$. For each $P, P' \in \mathscr{P}$, $\mu \in \text{supp}(\tau_P)$, and $\omega \in P'$, if $\omega \in \text{supp}(\mu)^*$, then $\lambda_{P'} \geq \lambda_P$.*

So far I have obtained the condition under which S does not want to mimic other states, the remaining task is to prove that S would not deviate to *out-of-equilibrium* messages.

What is S's minimal payoff if he verifies an event $E$, regardless of whether it is on or off the equilibrium path? Given that R holds a belief consistent with $E$ and

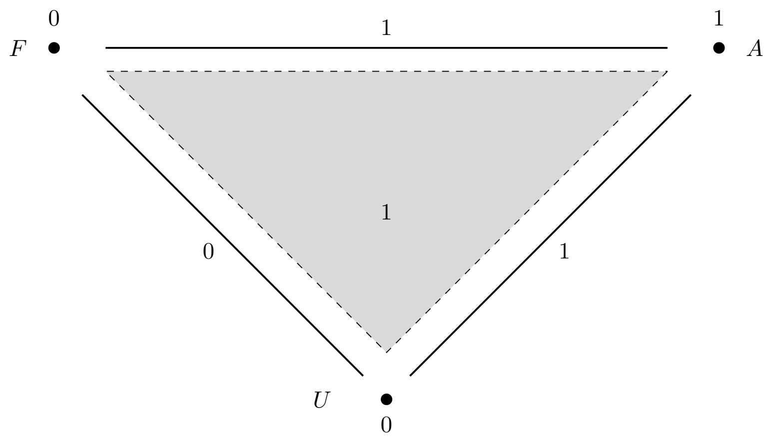



Figure 3: Lower Bound $b$ in Section 3.2

takes an optimal action, I denote this minimal payoff by

$$\pi(E) = \min\{U^S(a) \,|\, a \in r(\mu) \text{ for some } \mu \text{ with } \operatorname{supp}(\mu) \subseteq E\}.$$

When the state is $\omega$, S can guarantee himself a payoff $g(\omega) = \max\{\pi(E) \,|\, E \in \mathscr{F}(\omega)\}$ by providing information to verify a certain event. Hence, $g(\omega)$ serves as the *lower bound for S's equilibrium payoff at $\omega$*. Recall in the example of Section 3.2, a finance type can at least prove that he is skilled, but the firm might still reject his application, so $g(A) = 0$. A accounting type can identify himself with a certificate, in this case the firm would make an offer $M$, so $g(S) = 1$. For the unskilled, he can prove nothing, so $g(U) = 0$.

Generally, suppose at $\omega$, S sends a message $m_E$ ($\omega \in E$), which induces a belief $\mu = \phi(m_E)$ and a payoff $\lambda(\mu)$. His payoff should be no lower than the lower bound, so that $\lambda(\mu) \geq g(\omega)$. In addition, this inequality holds for all $\omega \in \operatorname{supp}(\mu)$, and state-independent preferences imply that the *lower bound for S's equilibrium payoff conditional on the realization of $\mu$* is $b(\mu) = \max\{g(\omega) \,|\, \omega \in \operatorname{supp}(\mu)\}$.

**Lemma 4.** *For each PBE outcome $(\tau, \lambda)$ and each $\mu \in \operatorname{supp}(\tau)$, $\lambda(\mu) \geq b(\mu)$.*

Lemma 4 suggests that any value point in $V$ below the lower bound $b(\cdot)$ should not be considered as an equilibrium outcome. Figure 3 presents the lower bound function in the example in Section 3.2. One feature of $b$ is that in the relative interior of each face of the belief simplex, $b$ takes on the same value.

As above, I have listed all conditions a PBE outcome has to satisfy, and I will show that these conditions are also sufficient for a PBE outcome. Finally, Theorem 1 summarizes these results and presents the characterization of PBE outcomes.

**Theorem 1.** $(\tau, \lambda)$ *is a PBE outcome if and only if there is a partition* $\mathscr{P}$ *governing* $\tau$ *such that:*

1. *For each* $\mu \in \operatorname{supp}(\tau)$, $\lambda(\mu) \in V(\mu)$ *and* $\lambda(\mu) \geq b(\mu)$.
2. *For each* $P \in \mathscr{P}$ *and* $\mu \in \operatorname{supp}(\tau_P)$, $\lambda(\mu) = \lambda_P$.
3. *For each* $P, P' \in \mathscr{P}$, $\mu \in \operatorname{supp}(\tau_P)$, *and* $\omega \in P'$, *if* $\omega \in \operatorname{supp}(\mu)^*$, *then* $\lambda_{P'} \geq \lambda_P$.

The first condition reflects the requirement of Lemma 4, that the sender's equilibrium payoff should be higher than what he can guarantee himself from presenting evidence. The second condition says that the state space is partitioned into equivalence classes and the information policy splits into conditional information policies in each partition member. The third condition says that if, in one partition member, a state is indistinguishable from the support of a belief in another partition element, S can mimic this information at the state. That means his payoff in the first partition member should be higher.

Here I give a sketch of the proof and postpone the details until Appendix B.2. The necessity of the conditions has been discussed previously in this section, so I restrict our attention to sufficiency.

These conditions are sufficient because whenever they are satisfied, I can construct a PBE. The key is using maximal weak evidence for $\operatorname{supp}(\mu)$ to induce each belief $\mu \in \operatorname{supp}(\tau)$. If S sends an *off-path* message $m_E$, I require that R holds a consistent belief and takes an optimal action that gives S payoff $\pi(E)$, which imposes severe punishment for deviation. Since R's optimization and the consistency of the belief system are relatively easy to prove, I will focus on constraints on S. Condition 1 guarantees that S's equilibrium payoff is so high that he would not like to send any *off-path* message. Condition 2 implies that S's payoffs are indifferent between every state in one partition member. Finally, whenever S is able to mimic some state outside his partition member, condition 3 implies that it is not profitable for him to do so. Therefore, in every state, S would neither deviate to *off-path* messages nor to equilibrium messages used in other states, so he will conform to the equilibrium strategy.

# 6 Full Verifiability

Full verifiability, which means that S has the ability to verify *any* true event, is the property of the message structure studied in Milgrom (1981) and Grossman (1981).

One situation that fits this assumption arises when every piece of information can be verified costlessly and there is an institution to enforce anti-fraud laws. The punishment for lying is so severe that S would not like to break the rule.

**Definition 6.** *A message structure satisfies* full verifiability *if for each event* $E \subseteq \Omega$, $\mathscr{M}(E)$ *is nonempty.*

In their settings, not only an equilibrium with full information exists, but also full information is the unique equilibrium outcome. This is called the unraveling result. The reasoning is straightforward. In the highest state, S will definitely disclose the true state; then in the second highest state, he cannot pretend to be the best state but can at least differentiate from lower states. Recursively, all states unravel from highest to lowest.

Nevertheless, under general preferences, even though full verifiability still guarantees the existence of a fully revealing equilibrium (Hagenbach, Koessler, and Perez-Richet, 2014), there could be other partially revealing equilibria. In Section 6.1, I provide a characterization as a corollary of Theorem 1. In Section 6.2, I reexamine the uniqueness of fully revealing equilibrium, and find that the conditions for the unraveling result in Milgrom (1981, 2008) can be relaxed to some extent.

## 6.1 Characterization

Full verifiability admits the strongest form of verification. Now that S can disclose the true state, the lower bound for the sender's equilibrium payoff at each $\omega$ is $g^f(\omega) = \min V(\delta(\omega))$. That is because any other evidence $m_F$, s.t. $\omega \in F$, cannot preclude the belief $\delta(\omega)$. So the worst outcome from presenting $m_F$, i.e., $\pi(F)$, is lower than truth-telling, i.e., $\pi(\{\omega\})$. Accordingly, the lower bound for the equilibrium payoff for S conditional on the realization of a belief $\mu$ becomes $b^f(\mu) = \max_{\omega \in \mathrm{supp}(\mu)} g^f(\omega)$.

**Lemma 5.** *Under a message structure with full verifiability, for each PBE outcome* $(\tau, \lambda)$ *and each* $\mu \in \mathrm{supp}(\tau)$, $\lambda(\mu) \geq b^f(\mu)$.

Condition 3 of Theorem 1 holds automatically. Because any event can be verified, the maximal weak evidence for each $E$ is $m_E$, i.e., $E^* = E$. So the situation where an indistinguishable state lies in other partition elements does not happen. Given the satisfaction of condition 3, the characterization is left with the first two conditions.

**Corollary 1.** *Under a message structure with full verifiability,* $(\tau, \lambda)$ *is a PBE outcome if and only if there is a partition* $\mathscr{P}$ *governing* $\tau$ *such that:*

1. *For each* $\mu \in \text{supp}(\tau)$*,* $\lambda(\mu) \in V(\mu)$ *and* $\lambda(\mu) \geq b^f(\mu)$*.*

2. *For each* $P \in \mathscr{P}$ *and* $\mu \in \text{supp}(\tau_P)$*,* $\lambda(\mu) = \lambda_P$*.*

Another implication of full verifiability is that to reveal the true state is an option S can return to any time. Hence, in each state $\omega$, the lowest payoffs S can receive in a fully revealing equilibrium, $g^f(\omega)$, are the lower bounds for his payoffs in any partially revealing equilibria. In this sense, S prefers partially revealing equilibria to fully revealing equilibria.

**Corollary 2.** *In every partially revealing equilibrium, suppose S's payoff at* $\omega \in \Omega$ *is* $t(\omega) \in \mathbb{R}$*, then I can find a fully revealing equilibrium where S gets paid no higher than* $t(\omega)$ *at* $\omega$*,* $\forall \omega$*.*

## 6.2 Revisiting the "Unraveling" Result

Milgrom (1981, 2008) prove the unraveling result under restrictive conditions. Milgrom (1981) assumes specific functional forms of S and R's payoffs; Milgrom (2008) relaxes the conditions but still needs a property of "single crossing." In this subsection, I further drop the "single crossing" condition and show that monotonicity and diminishing marginal utility suffice to guarantee full disclosure. In this subsection, I assume that $A \subseteq \mathbb{R}$ is a closed interval.

**Definition 7.** $u^S$ *is* monotone *if* $u^S$ *is strictly increasing in* $a$*.*

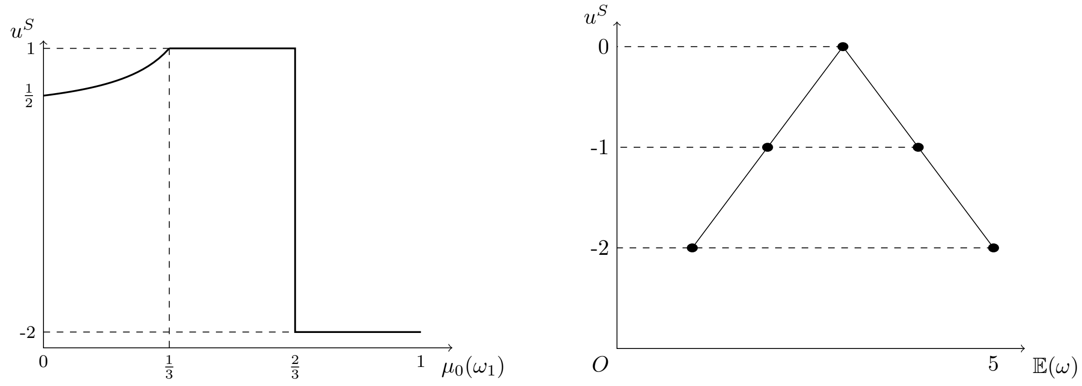
**Definition 8.** $u^R$ *satisfies* diminishing marginal utility *if for each* $\omega \in \Omega$*,* $u^R(\cdot,\omega)$ *is twice continuously differentiable and* $u^R_{11} < 0$ *for each* $a \in A$*.*

Diminishing marginal utility implies that R has a unique optimal response $r(\mu)$ to any belief $\mu$, and the optimal action is *pure*, i.e., $r(\mu) \in A$. Also, S's value correspondence can be regarded as a function of belief, $V(\mu) = u^S(r(\mu))$. These two observations will greatly simplify our analysis; in this section I will treat $r$ and $V$ as functions.

Then I introduce the definition of *full disclosure*.

**Definition 9.** *The equilibrium outcome is of* full disclosure *if for each* $\omega \in \Omega$ *and* $m \in \text{supp}(\sigma(\omega))$*,* $\rho(m) = r(\delta(\omega))$*.*

Full disclosure means that R has extracted all useful information with which she can make the optimal decision. All fully revealing equilibria are of full disclosure,




(a) Example 1

(b) Example 2

Figure 4: Failure of Unraveling

but the converse is not true. It is possible that even if R cannot distinguish between two states, full disclosure is still achieved. For example, if there are $\omega, \omega' \in \Omega$ s.t. $r(\delta(\omega)) = r(\delta(\omega'))$, Lemma 6 shows that as long as R knows the true state is contained in $\{\omega, \omega'\}$, she has a unique optimal choice. In this case, I still call the equilibrium outcome a full disclosure outcome where any other states than these two states are fully revealed.

**Lemma 6.** *If for each $\omega \in E \subseteq \Omega$, $r(\delta(\omega)) = \bar{r}$, then for each $\mu \in \Delta(E)$, $r(\mu) = \bar{r}$.*

Below I formalize the main result of this section.

**Theorem 2.** *When the message structure is of full verifiability and preferences satisfy monotonicity and diminishing marginal utility, every PBE outcome is of full disclosure.*

Let me give an intuition of the proof in the case when the receiver has distinctly different optimal responses in each state. (This idea applies to the general case.) By Lemma 5, if for a given $\mu \in \Delta(\Omega)$, $V(\mu)$ is less than $b^f(\mu)$, then $\mu$ cannot be a candidate for a belief induced by equilibrium strategy. Using this argument, I investigate the property of $V(\mu)$ and show that it has a "monotonic" functional form, in the sense that whenever the relative probability of a state associated with a higher optimal receiver response increases, the value of $V$ increases as well. It implies that for each non-degenerate belief $\mu \in \Delta(\Omega) \backslash \{\delta(\omega)\}_{\omega \in \Omega}$, I have $V(\mu) < b^f(\mu)$. Therefore, in equilibrium, the only possibility is that the information policy is supported on the extremes of the belief space, leading to full disclosure as the unique outcome.

When the game does not satisfy the above two assumptions, the unraveling result may not hold. Below I provide two examples to illustrate. The first example relaxes diminishing marginal utility and preserves monotonicity, and the second example relaxes monotonicity and preserves diminishing marginal utility. In both examples, I assume that the message structure is of full verifiability.

**Example 1**: Convex payoff function

The state space is binary $\Omega = \{\omega_0, \omega_1\}$ and the action set is an interval $A = [-2, 1]$; S's payoff function is $u^S(a) = a$; R's payoff function is concave in $\omega_0$, $u^R(a, \omega_0) = -(\frac{1}{2} - a)^2$, but convex in $\omega_1$, $u^R(a, \omega_1) = a^2$. The value function of $S$ has been shown in Figure 4(a). Notice that when $\mu_0(\omega_1) \leq \frac{2}{3}$, there is a pooling equilibrium where in both states S is paid higher than if he reveals the true states.

**Example 2**: Absolute difference

The state space is discrete real numbers $\Omega = \{1, 2, 3, 4, 5\}$ and the action set is an interval $A = [1, 5]$. S's payoff function is not monotonic, instead, it is equal to the absolute difference from 3, $u^S(a) = -|a - 3|$. R's payoff function is strictly concave $u^R(a, \omega) = -(a - \omega)^2$. With the quadratic loss function, R intends to estimate the value of $\omega$. Figure 4(b) illustrates S's payoff as a function of R's expectation of the true state, $\mathbb{E}(\omega)$. There is always a partially revealing equilibrium where each pair of states symmetric around 3 pool together, giving S a payoff higher than the fully revealing equilibrium.

# 7 Linear Disclosure

## 7.1 Characterization

In this section, I turn to study the case of *linear disclosure*, in which states are linearly ordered according to how many messages are available. A superior state has a message set that contains that of an inferior set.

**Definition 10.** *A message structure of* linear disclosure *if there is an order $\prec$ over $\Omega$ such that $M(\omega) \subseteq M(\omega')$ if $\omega \prec \omega'$.*

Without loss I relabel the states $\Omega = \{\omega_1, \ldots, \omega_{|\Omega|}\}$ to reflect the order of $\prec$, that is, $\omega_i \prec \omega_{i+1}$ for any $i$. Note that there could be ties, so there may be more than one way of ranking. For Corollary 3, it does not matter, but it will affect the results in Section 7.2, and there I will impose an additional constraint to address the issue of ties.

Let $E_i = \{\omega \in \Omega \,|\, \omega_i \prec \omega\}$ be the event that contains those states superior to $\omega_i$. Because S cannot distinguish a state from more superior states, $E_i$ is the most precise information S can verify in $\omega_i$. In particular, all the events S can verify in state $\omega_i$ are $\{E_j \subset \Omega \,|\, \omega_j \prec \omega_i\}$. Define $g^o(\omega_i) = \pi(E_i)$ and the lower bound function by $b^o(\mu) = \max_{\omega_i \in \text{supp}(\mu)} g^o(\omega_i)$.

**Lemma 7.** *In a linear disclosure model, for each PBE outcome $(\tau, \lambda)$ and each $\mu \in \text{supp}(\tau)$, $\lambda(\mu) \geq b^o(\mu)$.*

The partition members are linearly ordered in parallel with the disclosure order. Define an *ordered partition* $\mathscr{P}^o = \{P_1, \ldots, P_l, \ldots, P_L\}$ that is divided by several thresholds. Let $\omega_{n_l}$ denote the largest state within $P_l$, then each $P_l$ can be written as $\{\omega_{n_{l-1}+1}, \ldots, \omega_{n_l}\}$ $(n_0 = 0)$. I always make a partition in a way that the states in higher segment strictly dominate those in lower segments. That is, for each $l$, $\omega_{n_l+1} \succ \omega_{n_l}$ but $\omega_{n_l} \not\succ \omega_{n_l+1}$. Write the conditional information policy in $P_l$ as $\tau_l$ and the associated equilibrium payoff for S as $\lambda_l$.

As compared to Theorem 1, I can reduce condition 3 to that S's payoff in superior states should be higher than inferior states.

**Corollary 3.** *In a linear disclosure game, $(\tau, \lambda)$ is a PBE outcome if and only if there is an ordered partition $\mathscr{P}^o$ governing $\tau$ such that:*

1. *For each $\mu \in \text{supp}(\tau)$, $\lambda(\mu) \in V(\mu)$ and $\lambda(\mu) \geq b^o(\mu)$.*
2. *For each $1 \leq l \leq L$ and $\mu \in \text{supp}(\tau_l)$, $\lambda(\mu) = \lambda_l$.*
3. *For each $1 \leq l \leq l' \leq L$, $\lambda_l \leq \lambda_{l'}$.*

## 7.2 An algorithm to generate a PBE

In this section, I provide an algorithm to generate an information policy that satisfies the conditions of Corollary 3. The basic idea is that in equilibrium, states lie in one element of the partition only when in higher states of the disclosure order S prefers to pool with lower states. So I start with finding an ordered partition where the maximal state of each element has the lowest value. Then I develop a recursive program to find "cheap talk" equilibria in partition members from lowest to highest segments.

Below I introduce the algorithm in detail. In this part I take the existence of a cheap talk equilibrium as given, and all the detailed explanations are postponed

**Algorithm 1** Constructing a PBE in linear disclosure

**Input:** $\{P_1^{(1)}, \ldots, P_{n_1}^{(1)}\}$

**Output:** Information policy $\tilde{\tau}$, comprising conditional information policies $\{\tilde{\tau}_t\}_{t=1}^{n_1-y}$

$n = n_1$

$k = 1$

$y = 0$

**while** $n > 0$ **do**

**if** $v(\mu_0(\cdot|P_1^{(k)})) < c_1^{(k)}$ **then**

Obtain $\tau_1$, which is a conditional information policy on $P_1^{(k)}$ such that $\lambda(\mu) \in V(\mu)$ and $\lambda(\mu) = c_1^{(k)}$, for each $\mu \in \mathrm{supp}(\tilde{\tau}_t)$.

$\{P_1^{(k+1)}, \ldots, P_{n-1}^{(k+1)}\} = \{P_2^{(k)}, \ldots, P_n^{(k)}\}$

$\tilde{\tau}_{n_1+1-n-y} = \tau_1$

**else if** $v(\mu_0(\cdot|P_1^{(k)})) \in [c_1^{(k)}, c_2^{(k)})$ **then**

Obtain $\tau_1$, which is a conditional information policy on $P_1^{(k)}$ that puts all weight on $\mu_0(\cdot|P_1^{(k)})$.

$\{P_1^{(k+1)}, \ldots, P_{n-1}^{(k+1)}\} = \{P_2^{(k)}, \ldots, P_n^{(k)}\}$

$\tilde{\tau}_{n_1+1-n-y} = \tau_1$

**else if** $v(\mu_0(\cdot|P_1^{(k)})) \geq c_2^{(k)}$ **then**

$\{P_1^{(k+1)}, \ldots, P_{n-1}^{(k+1)}\} = \{P_1^k \cup P_2^{(k)}, \ldots, P_n^{(k)}\}$

$y = y + 1$

**end if**

$n = n - 1$

$k = k + 1$

**end while**

until Appendix D.3.

*Step 0*

Let $v$ be any selector of $V$. That is, $v$ is a single-valued function on $\Delta(\Omega)$ and $v(\mu) \in V(\mu)$, for any $\mu \in \Delta(\Omega)$. If there are ties in the disclosure order, i.e., $M(\omega_i) = M(\omega_j)$, then compare $v(\delta(\omega_i))$ and $v(\delta(\omega_j))$: if $v(\delta(\omega_i)) > v(\delta(\omega_j))$, put $\omega_i$ before $\omega_j$, so that $i < j$; if $v(\delta(\omega_i)) = v(\delta(\omega_j))$, the ranking is arbitrary. Let $c_i = \min_{j \geq i} v(\delta(\omega_j))$ represent the lowest payoff S can get in a state $\omega_j \succ \omega_i$. By definition, $c_i$ is increasing in $i$. Selecting states that share the same value of $c_i$ into one part, I obtain an ordered partition $\mathscr{P}^{(1)} = \{P_1^{(1)}, \ldots, P_{n_1}^{(1)}\}$. Denote by $\omega_i^{(1)}$ the maximal state in each $P_i^{(1)}$ and by $c_i^{(1)} = v(\delta(\omega_i^{(1)}))$ the corresponding minimum value of each partition element.

*Step $(k)$, $k = 1, \ldots, n_1$*

Suppose $\tilde{\tau}_1, \ldots, \tilde{\tau}_{t-1}$ have been defined, now I search for a conditional information policy on $P_1^{(k)}$.

*Case k-1*: If $v(\mu_0(\cdot|P_1^{(k)})) < c_1^{(k)}$, from Theorem 1 of Lipnowski and Ravid (2020), I can find a conditional information policy $\tilde{\tau}_t$ on $P_1^{(k)}$ such that $\lambda(\mu) \in V(\mu)$ and $\lambda(\mu) = c_1^{(k)}$, for each $\mu \in \text{supp}(\tilde{\tau}_t)$.

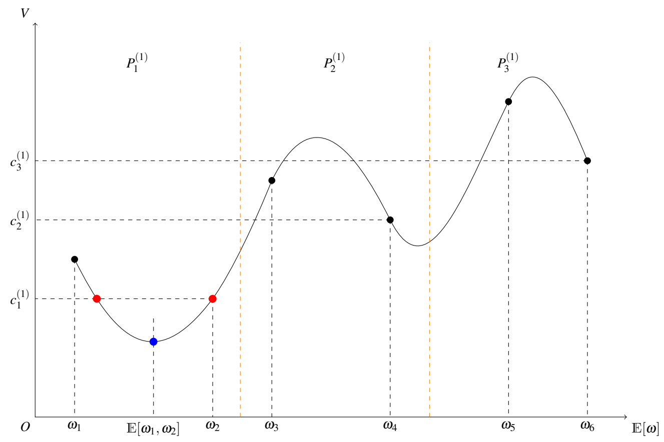
*Case k-2*: If $v(\mu_0(\cdot|P_1^{(k)})) \in [c_1^{(k)}, c_2^{(k)})$, let $\tilde{\tau}_1$ put all weight on $\mu_0(\cdot|P_1^{(k)})$.

In either case k-1 or case k-2, define an ordered partition $\mathscr{P}^{(k+1)} = \{P_1^{(k+1)}, \ldots, P_{n_{k+1}}^{(k+1)}\}$, where $n_{k+1} = n_k - 1$ and $P_j^{(k+1)} = P_{j+1}^{(k)}$, for any $j$. $\mathscr{P}^{(k+1)}$ is the same as $\mathscr{P}^{(k)}$ except for excluding the first element $P_1^{(k)}$. In addition, denote by $\omega_i^{(k+1)}$ the maximum state in each $P_i^{(k+1)}$ and by $c_i^{(k+1)} = v(\delta(\omega_i^{(k+1)}))$ the corresponding minimum value of each partition element.

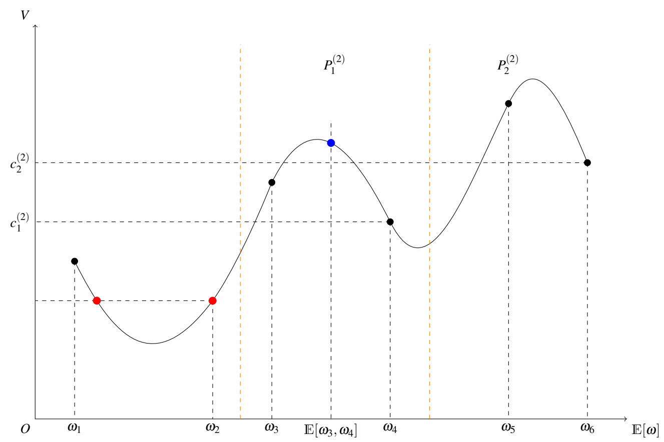
*Case k-3*: If $v(\mu_0(\cdot|P_1^{(k)})) \geq c_2^{(k)}$, the new ordered partition is defined as $\mathscr{P}^{(k+1)} = \{P_1^{(k+1)}, \ldots, P_{n_{k+1}}^{(k+1)}\}$, where $n_{k+1} = n^k - 1$, $P_1^{(k+1)} = P_1^{(k)} \cup P_2^{(k)}$, and $P_j^{(k+1)} = P_{j+1}^{(k+1)}$, for any $j > 1$. Also, $\omega_i^{(k+1)}$ and $c_i^{(k+1)}$ are defined as above accordingly.

This process ends after $n_1$ steps with a series of conditional information policies $\{\tilde{\tau}_t\}_{t=1}^T$, $T \leq n^1$, being generated. By construction, the *ex ante* distribution over $\{\tilde{\tau}_t\}_{t=1}^T$, $\tilde{\tau}$, is an information policy satisfying the conditions of Corollary 3. Therefore, it means that $\tilde{\tau}$ is a PBE outcome of the linear disclosure game.

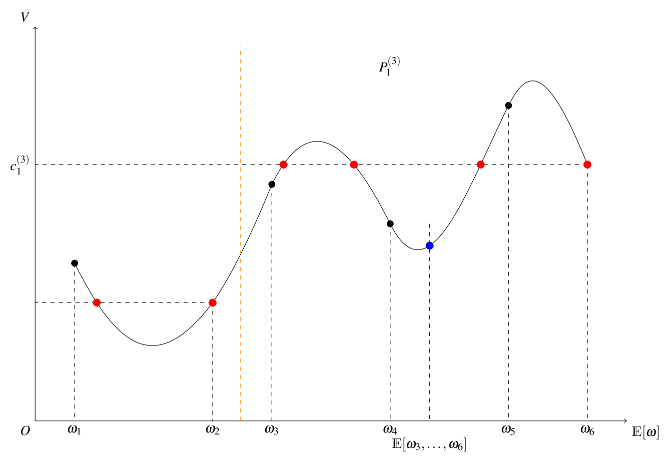
**Proposition 1.** *An information policy $\tilde{\tau}$ and the payoff distribution $\lambda$ generated through the above algorithm is a PBE outcome of the linear disclosure game.*




(a) $\mathscr{P}^{(1)}$




(b) $\mathscr{P}^{(2)}$




(c) $\mathscr{P}^{(3)}$

Figure 5: Illustration of how the algorithm generates a PBE

## 7.3 An application of the algorithm

I will use a general example to show how to use the algorithm to generate a PBE outcome. Suppose the unknown state is a real number from $\{\omega_1, \omega_2, \ldots, \omega_6\} \subseteq \mathbb{R}$. R's decision only depends on the expectation of random variable and she always has a unique optimal action. Therefore, S's value correspondence $V$ is single valued, as illustrated in Figure 5.

Based on S's value in each state, $\{V(\omega_i)\}_{i=1}^{6}$, first I partition the state space into three segments, $\mathscr{P}^{(1)} = \{\{\omega_1, \omega_2\}, \{\omega_3, \omega_4\}, \{\omega_5, \omega_6\}\}$, with $c_i^{(1)} = V(\omega_{2i})$, for $i = 1, 2, 3$. Consider that S pools $P_1^{(1)}$, then his value $V(\mathbb{E}[\omega_1, \omega_2]) < c_1^{(1)}$. This is case 1 (Section 7.2), so I can find a conditional information policy $\tilde{\tau}_1$ on $\mathscr{P}_1^{(1)}$ with constant associated payoffs equal to $c_1^{(1)}$. The conditional information policy and associated payoff are illustrated by the red points in Figure 5(a).

Second, I delete $P_1^{(1)}$ from $\mathscr{P}^{(1)}$ and obtain a new partition $\mathscr{P}^{(2)} = \{\{\omega_3, \omega_4\}, \{\omega_5, \omega_6\}\}$ with $c_i^{(2)} = V(\omega_{2(i+1)})$, $i = 1, 2$. Since $V(\mathbb{E}[\omega_3, \omega_4]) > c_2^{(2)}$, this situation lies in case 3. So I move on to the next step directly.

Third, I obtain $\mathscr{P}^{(3)}$ by merging $P_1^{(2)}$ and $P_2^{(2)}$, so that $\mathscr{P}^{(3)} = \{\omega_3, \omega_4, \omega_5, \omega_6\}$ and $c_1^{(3)} = V(\omega_6)$. Because $V(\mathbb{E}[\omega_3, \ldots, \omega_6]) < c_1^{(3)}$, it comes back to case 1 again. So I can find a conditional information policy $\tilde{\tau}_2$ on $\mathscr{P}^{(3)}$ with associated payoffs equal to $c_1^{(3)}$. Finally, the equilibrium outcome and partitional form are illustrated by the red points in Figure 5(c).

# 8 Example: Political Campaign

In this section, I consider a political campaign where an incumbent seeks re-election. There has been a literature on political economy that discusses incumbency advantage, i.e., the high re-election rate for an incumbent. In particular, Ashworth, De Mesquita, and Friedenberg (2019) argue that this phenomenon may have an informational drive. Because the public usually have more information about the incumbent than challengers, it is easier for the public to receive positive signals about the incumbent's characteristics, if he is believed to be competitive. Then, incumbency advantage only reflects high expectations on the incumbent in the first place. In the following example, I also discuss how information plays a role in the election, yet the information comes from the incumbent's strategic disclosure, instead of being generated exogenously.

Consider a political campaign in which an incumbent is running for re-election.

There are a continuum of voters $[0,1]$ who care about the incumbent's level of of competence and policy preference. Both competence level and policy preference are private information for the incumbent. The incumbent's level of competence, viz. $\theta$, may be high, $\bar{\theta}$, or low, $\underline{\theta}$. The incumbent may favor policy $p \in \{a,b\}$. Thus, the state space is $\Omega = \{\bar{\theta}, \underline{\theta}\} \times \{a,b\}$. There is a common prior $\mu_0$ on $\Omega$.

Voters are divided into two groups: $A$ and $B$. Group $A$ (resp. $B$) supports policy $a$ (resp. $b$). Let $[0,s)$ be the set of voters in group $A$ and $[s,1]$ be the set of voters in group $B$, where $0 < s < \frac{1}{2}$.[15] Also, suppose there is an outside option for voters, e.g., a challenger who competes with the incumbent in this election. If the challenger is elected, voters receive reservation payoff 0.

The incumbent produces a social outcome as a function of competence and his policy preference. A voter (from group $i$) cares about social outcome, and her preference over social outcome is described by a function $f_i(\theta,p)$. If the incumbent is incompetent, the social outcome is bad for the voter, irrespective of the social preference of the incumbent. So that $f_i(\underline{\theta},p) = -1$, for each $p \in \{a,b\}$. If the incumbent is competent, he produces a social outcome that he prefers, so that

$$f_i(\bar{\theta},a) = \begin{cases} w, & \text{if } i \in A, \\ -1, & \text{if } i \in B. \end{cases} \qquad f_i(\bar{\theta},b) = \begin{cases} -1, & \text{if } i \in A, \\ 1, & \text{if } i \in B. \end{cases}$$

where $w$ is the relative willingness of group $A$ to support policy $a$, as compared to the willingness of group $B$ to support policy $b$.

Suppose voters have heterogeneous preferences for incumbency, the payoff to a voter from group $i$ is $u_i^R = f_i + \epsilon_i$, where $\epsilon_i \sim U[-t_i, t_i]$ captures the heterogeneity.[16] To make sure that in $\bar{\theta}a$ (resp. $\bar{\theta}b$), the entire group $A$ (resp. $B$) will vote for the incumbent, I impose a constraint on the variance of $\epsilon_i$, $i = A,B$: $t_i < \min\{w,1\}$. So the random error will not affect the ordinal preferences of the voters.

Each voter chooses whether to vote for the incumbent, and her choice depends on her belief about the state. Let $\mu$ be a probability distribution over the state space. A voter (from group $i$) votes for the incumbent if and only if $U_i^R(\mu) \geq 0$, that is, $\mathbb{E}[f_i(\theta,p)\,|\,\mu] + \epsilon_i \geq 0$. Therefore, the expectation of the incumbent's vote share, i.e., the incumbent's value function, is $V(\mu) = s \cdot Pr(\epsilon_A \geq -\mathbb{E}[f_A(\theta,p)\,|\,\mu]) + (1-s) \cdot Pr(\epsilon_B \geq -\mathbb{E}[f_B(\theta,p)\,|\,\mu])$.

[15] The case of $s > \frac{1}{2}$ is symmetric and the similar results hold. The case of $s = \frac{1}{2}$ is not interesting because then the incumbent is indifferent between getting support from $A$ and $B$, and his information about his policy preference is unimportant.

[16] To avoid complex calculations, I assume uniform distribution of $\epsilon_i$. But allowing more general distributions will not lose the main insights.

The incumbent's goal is to maximize his vote share. To this end, he can reveal relevant information to influence the voters' belief about him. Suppose during the course of his governance, the incumbent has implemented many policies other than policies $a$ and $b$. If he is competent, his policies should have produced positive social outcomes, which he can reveal to the public to verify his competence. That is, there is message $m_{\bar{\theta}}$ that verifies $\{\bar{\theta}a, \bar{\theta}b\}$. Also, assume that he can credibly reveal his policy preference for policy $b$ by sending message $m_b$. But he cannot verify his preference for policy $a$.[17] The combination of $m_{\bar{\theta}}$ and $m_b$ verifies $\{\bar{\theta}b\}$, which is described by message $m_{\bar{\theta}b}$. If the incumbent keeps silent or prevaricates, his message is $m_\Omega$. Therefore, there are four types of messages: $m_{\bar{\theta}}$, $m_b$, $m_{\bar{\theta}b}$, and $m_\Omega$.[18]

A trivial equilibrium is where the incumbent sends $m_{\bar{\theta}b}$ in $\bar{\theta}b$, sends $m_{\bar{\theta}}$ in $\bar{\theta}a$, and sends $m_\Omega$ in $\underline{\theta}a$ and $\underline{\theta}b$. This equilibrium has an outcome of full disclosure, since each voter has a clear mind of whether the incumbent is the person she wants to vote for. From the incumbent's perspective, when he is incompetent, he will lose; when he is competent, he will receive support from the group which share the same policy preference with him, while drive away voters from the opposite group.

However, the fully revealing outcome may not be the best the incumbent could achieve. He may play another strategy that he is crystal clear about whether he is competent or not, but vague about his preferred policy. In what follows I will focus on this type of partially revealing equilibrium with one dimensional disclosure, denoted by $PRE-1$. I will discuss in what situations there exists a PRE-1 such that a competent incumbent ($\theta = \bar{\theta}$) gets strictly better off than in the fully revealing equilibrium. Before stating the result, I set $\mu_i = \mu_0(\theta = \underline{\theta})$, $\mu_a = \mu_0(\bar{\theta}a)$, and $\mu_b = \mu_0(\bar{\theta}b)$ and use $\pi = \frac{\mu_b}{\mu_a + \mu_b}$ to denote the relative probability that the incumbent supports policy $b$.

**Case 1:** $\epsilon_A = \epsilon_B \sim U[-t, t]$

When two groups have homogeneous random error and $\pi$ lies in a certain area, it is strictly better for a competent incumbent to withhold information. In his favorite PRE-1, the incumbent simply pools $\bar{\theta}a$ and $\bar{\theta}b$.

**Proposition 2.** *1. When $w > 1$ and $\pi \in [\frac{1+t}{2}, \frac{t+w}{1+w})$, there exists a PRE-1 in which the incumbent receives more votes than $(1-s)$ in both $\bar{\theta}a$ and $\bar{\theta}b$.*

[17] Even if he can verify $p = a$, the main results do not change.

[18] The results in this section hold under the message structure of full verifiability, as well.

2. *When $w \leq 1$, the incumbent cannot receive more votes than $(1-s)$ in $\bar{\theta}b$ and more than 0 in $\theta = \underline{\theta}$. When $\pi < \frac{1+t}{2}$, the fully revealing equilibrium is the unique equilibrium.*

Proposition 2 demonstrates that a PRE-1 strictly benefits a competent incumbent when the smaller group (A) have stronger willingness to support their policy than the larger group (B). The disparity in two groups' willingness creates different responsiveness to information, which the incumbent can leverage to increase his overall support. When the incumbent is thought to be leaning towards $B$, yet voters are not completely sure, he will not lose votes from his core supporters by being vague, while in the meantime he can lure voters from the other side.

**Case 2:** $\mu_0 = (\mu_i, \mu_a, \mu_b) = (\frac{1}{6}, \frac{1}{6}, \frac{2}{3})$, $s = \frac{4}{9}$, $w = \frac{3}{2}$, $\epsilon_A \sim U[-\frac{1}{2}, \frac{1}{2}]$, $\epsilon_B \sim U[-1, 1]$

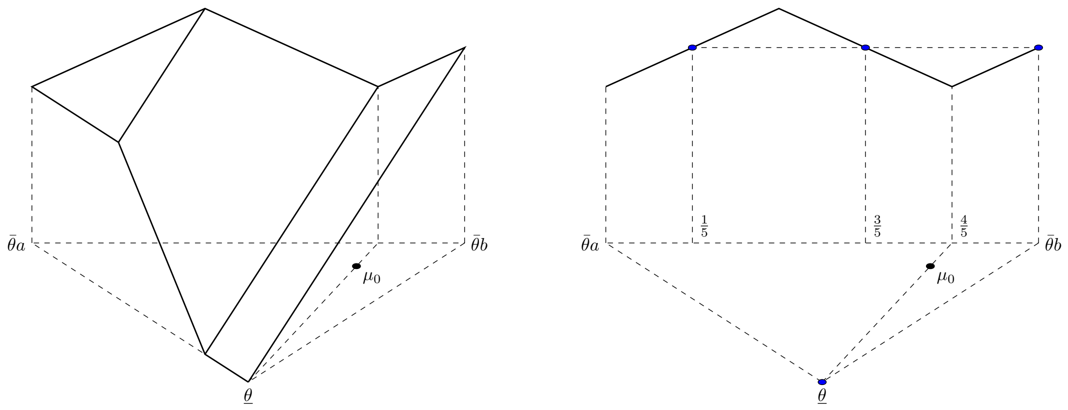
When the variance of $\epsilon_i$ varies across groups, to achieve the incumbent's preferred equilibrium may require that he plays mixed strategies. Here group $B$ is more heterogeneous than group $A$, so the change in group $A$'s support for the incumbent is faster. The total vote share as a function of beliefs is illustrated in Figure 6(a).

Figure 6(b) represents the incumbent's preferred PRE-1 outcomes. There is a continuum of equilibria that share a common feature that the information policies are supported on $(0,0,0)$, $(0, \frac{1}{5}, \frac{4}{5})$, $(0, \frac{3}{5}, \frac{2}{5})$, and $(0,0,1)$. One typical equilibrium is that the incumbent sends $m_{\bar{\theta}}$ in $\bar{\theta}a$, and randomize between $m_{\bar{\theta}b}$ and $m_{\bar{\theta}}$ with probabilities $\frac{15}{16}$ and $\frac{1}{16}$ in $\bar{\theta}b$. Then after $m_{\bar{\theta}}$ the voters' belief is updated to $(0, \frac{3}{5}, \frac{2}{5})$, so the incumbent will get support from half of group $A$ and more than half of group $B$; after $m_{\bar{\theta}b}$ he will have the full support of group $B$.

In those equilibria, a competent incumbent, no matter whether he supports policy $a$ or policy $b$, his vote share is equal to $\frac{5}{9}$. As compared to his performance in the fully revealing equilibrium, where he would get $\frac{4}{9}$ in $\bar{\theta}a$ and $\frac{5}{9}$ in $\bar{\theta}b$, his payoff is strictly improved when he supports policy $a$. Different from *Case 1*, the incumbent benefits from luring voters from group $B$ in *Case 2*.

# 9 Conclusion

Evidence speaks for itself, but it can also be used as tools for persuasion. When a sender makes strategic moves, his message is a mix of presentation of evidence and randomization over messages. Hence, in persuasive disclosure, information is




(a) Vote Function

(b) Incumbent's Favorite PRE-1

Figure 6: Heterogeneous Errors

segmented into mutually exclusive categories; and in each category, the sender persuades the receiver as if he was playing a cheap talk game. Therefore, the main result of this paper reveals an intriguing relation between a disclosure model and cheap talk: information disclosure is essentially partitional cheap talk.

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# Appendix

## A Section 4

### A.1 Proof of Lemma 1

"$\supseteq$" For each $m \in \bigcup_{E \in \mathscr{F}(\omega)} \mathscr{M}(E)$, there is $F \subseteq \Omega$, s.t. $\omega \in F$ and $m \in \mathscr{M}(F)$. By the definition of $\mathscr{M}(F)$, $m \in M(\omega)$.

"$\subseteq$" For each $m \in M(\omega)$, let $F = \{\omega' \in \Omega : m \in M(\omega')\}$. $F$ is nonempty since $\omega \in F$. $m$ verifies $F \ni \omega$. So that $m \in \mathscr{M}(F) \subseteq \bigcup_{E \in \mathscr{F}(\omega)} \mathscr{M}(E)$.

### A.2 Proof of Lemma 2

Because $\mathscr{M}^*(E)$ is nonempty, $\mathscr{W}(E)$ is nonempty, as well. Without loss of generality, write $\mathscr{W}(E) = \{K_1, \ldots, K_n\}$. By the definition of $\mathscr{W}$, I know that for each $i$, $K_i \supseteq E$, and there exists $m_{K_i} \in \mathscr{M}^*(E)$ that verifies $K_i$. So that for each $i$, $\mathscr{M}(K_i)$ is nonempty. By strong normality, $\mathscr{M}(K_1 \cap K_2)$ is nonempty, $\mathscr{M}(K_1 \cap K_2 \cap K_3)$ is nonempty, ..., $\mathscr{M}(\bigcap_{K \in \mathscr{W}(E)} K)$ is nonempty.

## B Section 5

### B.1 Proof of Lemma 4

Prove by contradiction. If in a PBE $\mathcal{E} = (\sigma, \rho, \phi)$, $m$ is an equilibrium message that induces $\phi(m) = \mu$ and $U^S(\rho(m)) = \lambda(\mu)$, but $U^S(\rho(m)) < b(\phi(m))$. That means there is $\omega \in \text{supp}(\phi(m))$ such that $\sigma(\omega)(m) > 0$ and $U^S(\rho(m)) < g(\omega)$. By the definition of $g(\omega)$, it means that there is $E \in \mathscr{F}(\omega)$ such that $U^S(\rho(m)) < \pi(E)$. However, by the definition of $\pi(E)$, $U^S(\rho(m_E)) \geq \pi(E) > U^S(\rho(m))$. So the sender will deviate to $m_E$ from $m$ in $\omega$.

### B.2 Proof of Theorem 1

(Necessity)

Suppose $\mathcal{E} = (\sigma, \rho, \phi)$ is a PBE and $\tau$ is the information policy associated with $\sigma$. Below I prove the satisfaction of the three conditions one by one.

1. From the optimization of R's response, I know that $(\mu, \lambda(\mu)) \in graph(V)$. Also by Lemma 4, $\lambda(\mu) \geq b(\mu)$ for each $\mu \in \text{supp}(\tau)$, so that the first condition is satisfied.

2. For each $\omega, \omega' \in \Omega$, if there exists one message $m$ such that $m \in \text{supp}(\sigma(\omega)) \cap \text{supp}(\sigma(\omega'))$, for each $m' \in \text{supp}(\sigma(\omega))$ and each $m'' \in \text{supp}(\sigma(\omega'))$, $U^S(\rho(m)) = U^S(\rho(m')) = U^S(\rho(m''))$. Because S's payoff is state-independent and S must randomizes between messages that bring him equal payoffs. This equivalence relation between states constructs equivalence classes that form a partition $\mathscr{P}$ of $\Omega$.

Because any states that commonly use at least one message are in one partition element of $P$, any message can only be used by states belonging to one partition element. That means the support of the posterior belief updated from any equilibrium message must lie in one equivalence class. Hence, $\mathscr{P}$ governs $\tau$, i.e., $\tau$ splits into a collection of conditional information policies in each partition element, $\{\tau_P\}_{P \in \mathscr{P}}$.

For each $P \in \mathscr{P}$ and each $\mu, \mu' \in \text{supp}(\tau_P)$, $\mu$ and $\mu'$ are derived from messages sent by states in one equivalence class, so that $\lambda(\mu) = \lambda(\mu') = \lambda_P$.

3. For each $P, P' \in \mathscr{P}$, $\mu \in \text{supp}(\tau_P)$, and $\omega \in P' \cap \text{supp}(\mu)^*$, there exists $m_E$ such that $\phi(m_E) = \mu$ and for each $\omega \in \text{supp}(\mu)$, $m_E \in M(\omega)$ and $\sigma(\omega)(m_E) > 0$. Therefore, $\text{supp}(\mu) \subseteq E$. Be Lemma 2, there is $\text{supp}(\mu)^* \subseteq E$, so that $\omega \in \text{supp}(\mu)^* \subseteq E$. That means $m_E \in M(\omega)$. Since in $\omega' \in P'$, S can mimic information in $P$, S's payoff in $\omega$, $\lambda_{P'}$, should be no lower than $U^S(\rho(m_E)) = \lambda_P$. Otherwise, S will deviate to sending $m_E$ in $\omega$.

(Sufficiency)

Construct an equilibrium $\mathcal{E} = (\sigma, \rho, \phi)$ such that:

(i) $\sigma$ generates $\tau$ under the constraint that the equilibrium messages S uses to induce each $\mu \in \text{supp}(\tau)$ are from the set $\mathscr{M}(\text{supp}(\mu)^*)$.

(ii) For each on-path message $m$ with $\phi(m) = \mu$, let R choose an action $\rho(m) \in$

$r(\mu)$ such that $U^S(\rho(m)) = \lambda(\mu)$. For any off-path message $m$, let R choose $\rho(m) \in \arg\min_{a \in r(\phi(m))} U^S(a)$.

(iii) Bayes' rule applies to any message sent with positive probability. While for any unsent message $m_E$, assign a belief $\phi(m_E)$ so that $\min V(\phi(m_E)) = \pi(E)$.

This construction is feasible for three reasons. First, for each $\mu \in \text{supp}(\tau)$, there are at least cheap talk messages that can be used to induce it. Second, by Lemma 2, $\mathscr{M}(\text{supp}(\mu)^*) \neq \emptyset$. Third, because $\lambda(\mu) \in V(\mu)$, R has corresponding strategies to achieve the outcomes.

By construction, it is straightforward that R optimizes and the belief system is consistent, so I focus on showing that S has no incentive to deviate in any state.

For each $P \in \mathscr{P}$ and $\omega \in P$, the set of messages S can deviate to, i.e., $M(\omega) \setminus \text{supp}(\sigma(\omega))$, can be divided into three parts. The first part contains the equilibrium messages used in other states in $P$, the second part the equilibrium messages used in $P' \in \mathscr{P}$ ($P' \neq P$), and the third part the out-of-equilibrium messages in $M(\omega)$.

First, by condition 2, S's payoffs in any state within $P$ are the same, so he would not like to deviate to other equilibrium messages used in $P_l$. Second, if in $\omega \in P$, S can send a message $m_E$ which is being used in $\omega' \in P'(P' \neq P)$, by condition (i), I have $\omega \in \text{supp}(\phi(m_E))^*$, then by condition 3, $\lambda_P \geq \lambda_{P'}$. So he will not deviate to equilibrium messages used in $P'$. Finally, if S deviates to an out-of-equilibrium message $m_F$ ($\omega \in F$), by conditions (ii) and (iii), his payoff is $\pi(F) \leq g(\omega) \leq \lambda_P$. So S will not deviate to out-of-equilibrium messages.

Therefore, S would like to conform to the equilibrium strategy.

# C Section 6

## C.1 Proof of Lemma 5

Prove by contradiction. If in a PBE $\mathcal{E} = (\sigma, \rho, \phi)$, $m$ is an equilibrium message that induces $\phi(m) = \mu$ and $U^S(\rho(m)) = \lambda(\mu)$, but $U^S(\rho(m)) < b^f(\phi(m))$. That means there is $\omega \in \text{supp}(\phi(m))$ such that $\sigma(\omega)(m) > 0$ and $U^S(\rho(m)) < g^f(\omega) = \min V(\delta(\omega)) \leq U^S(\rho(m_{\{\omega\}}))$. So S will deviate to $m_{\{\omega\}}$ from $m$ in $\omega$.

## C.2 Proof of Corollary 1

It suffices to show that the conditions are equivalent to those in Theorem 1. Condition 1 is implied by Lemma 5. Condition 2 coincides in both places. The remaining task is to show that the situation in condition 3 does not occur in the case of full verifiability. When the message structure is of full verifiability, for each $E$, $\mathscr{M}(E) \neq \emptyset$, and $m_E$ is maximal weak evidence for $E$. In other words, $E^* = E$. Then, for each $\mu \in \text{supp}(\tau)$ s.t. $\text{supp}(\mu) \subseteq P \in \mathscr{P}$, $\text{supp}(\mu)^* \subseteq P$. So there is no $\omega \in \text{supp}(\mu)^*$, such that $\omega \in P' \neq P$.

### C.3 Proof of Lemma 6

Since $\mu = \sum_{\omega \in E} \mu(\omega)\delta(\omega)$, then I have $U^R(a,\mu) = \sum_{\omega \in E} \mu(\omega) U^R(a, \delta(\omega))$. Because $\bar{r}$ maximizes each $U^R(\cdot, \delta(\omega)) = u^R(\cdot, \omega)$, for any $\omega \in E$, $\bar{r}$ also maximizes their convex combination $U^R(\cdot, \mu)$.

### C.4 Proof of Theorem 2

One immediate result following diminishing marginal utility is that $U^R(a,\mu) = \sum_{\omega \in \Omega} \mu(\omega) u^R(a,\omega)$ is strictly concave for any $\mu \in \Delta(\Omega)$, which satisfies the assumption of "single-peakedness" proposed in Hart, Kremer, and Perry (2017). Since the framework in this section lies in the model of Hart, Kremer, and Perry (2017), I can appeal to their result to have a useful property: the "in-betweenness" property.

**Definition 11.** *The preferences have the in-betweenness property if for any set of beliefs $\{\mu_1, \ldots, \mu_m\} \subset \Delta(\Omega)$, and for any $\tilde{\mu}$ in the convex hull of $\{\mu_1, \ldots, \mu_m\}$, denoted by $\tilde{\mu} \in conv(\{\mu_1, \ldots, \mu_m\})$, I have*

$$\min_{1 \le i \le m} r(\mu_i) \le r(\tilde{\mu}) \le \max_{1 \le i \le m} r(\mu_i) \tag{1}$$

*Because $V(\mu) = u^S(r(\mu))$, it follows from monotonicity that*

$$\min_{1 \le i \le m} V(\mu_i) \le V(\tilde{\mu}) \le \max_{1 \le i \le m} V(\mu_i) \tag{2}$$

**Lemma 8.** *Under the assumptions of monotonicity and diminishing marginal utility, the preferences have the in-betweenness property.*

Then I will show that the value function $V(\mu)$ changes monotonically with the belief $\mu$.

**Lemma 9.** *For any $\mu, \mu' \in \Delta(\Omega)$ s.t. $V(\mu) < V(\mu')$ and $\lambda \in (0,1)$, $V(\mu) < V((1-\lambda)\mu + \lambda\mu') < V(\mu')$.*

*Proof.* Let $\mu'' = (1-\lambda)\mu + \lambda\mu'$, for some $\lambda \in [0,1]$. $r(\mu'')$ is the solution to the equation $U_1^R(a,\mu'') = 0$ as below

$$(1-\lambda)U_1^R(a,\mu) + \lambda U_1^R(a,\mu') = 0 \tag{3}$$

Because $u^S(\cdot,\omega)$ is twice continuously differentiable, $U_{11}^R$ is continuous, so I can appeal to the Implicit Function Theorem. By the Implicit Function Theorem, the derivative of the solution of $a$ with respect to $\lambda$ is

$$\frac{dr(\cdot)}{d\lambda} = \frac{U_1^R(a^*,\mu) - U_1^R(a^*,\mu')}{(1-\lambda)U_{11}^R(a^*,\mu) + \lambda U_{11}^R(a^*,\mu')} \tag{4}$$

where $a^*$ is the solution of Eq.(3). Because of Lemma 8, $a^* \in [r(\mu), r(\mu')]$. Because $U^R$ is strictly concave in $a$, $U_1^R(a^*,\mu)$ is non-positive in $[r(\mu), r(\mu')]$, while $U_1^R(a^*,\mu')$ is non-negative in $[r(\mu), r(\mu')]$, and they cannot be zero at the meantime. That means when $a^* \in [r(\mu), r(\mu')]$, $U_1^R(a^*,\mu) - U_1^R(a^*,\mu') < 0$. In addition, the denominator of the right hand side of Eq.(3) is negative from diminishing marginal utility, so the derivative $dr(\cdot)/d\lambda$ is strictly positive for any $\lambda \in [0,1]$. So that for $\lambda \in (0,1)$, $r(\mu) < r((1-\lambda)\mu + \lambda\mu') < r(\mu')$. Since $V(\mu) = u^S(r(\mu))$ and $u^S$ is strictly increasing, it is true that $V(\mu) < V((1-\lambda)\mu + \lambda\mu') < V(\mu')$. □

Consider R's optimal actions of each state $\{r(\delta(\omega))\}_{\omega\in\Omega}$, I call $\omega \in E \subseteq \Omega$ a *maximal* state in $E$ if $r(\delta(\omega)) = \max_{\omega'\in E} r(\delta(\omega'))$. A maximal state is one of S's favorite states in $E$. Note there might be ties, so I denote by $E^{max}$ the set of all maximal states in $E$. This preference for maximal states also extends to probability distributions. I will show that the belief in $\Delta(E^{max})$ leads to a higher action than other beliefs in $\Delta(E)\backslash\Delta(E^{max})$.

**Lemma 10.** *For any $E \subseteq \Omega$, $\mu \in \Delta(E^{max})$, and $\mu' \in \Delta(E)\backslash\Delta(E^{max})$, $V(\mu) > V(\mu')$.*

*Proof.* If $E^{max} = E$, by Lemma 6, $\Delta(E^{max}) = \Delta(E)$ and $\Delta(E)\backslash\Delta(E^{max}) = \emptyset$, so the result is trivially true. Otherwise, when $E^{max} \neq E$, there is $\lambda \in (0,1]$, s.t. $\mu' = (1-\lambda)\mu'(\cdot|E^{max}) + \lambda\mu'(\cdot|E\backslash E^{max})$. By Lemma 8, $V(\mu'(\cdot|E\backslash E^{max}) \leq \max_{\omega\in E\backslash E^{max}} V(\mu_\omega)$. By Lemma 6 and the definition of $E^{max}$, $V(\mu'(\cdot|E^{max})) = \max_{\omega\in E\backslash E^{max}} V(\mu_\omega) > \max_{\omega\in E\backslash E^{max}} V(\mu_\omega) \geq V(\mu'(\cdot|E\backslash E^{max})$. Then by Lemma 6 and Lemma 9, $V(\mu) = V(\mu'(\cdot|E^{max})) > V(\mu')$. □

Next, I will show that the value function falls beneath the lower bound $b^f(\cdot)$ except in beliefs that are supported on their own maximal states.

**Proposition 3.** *For any $\mu$ such that* $\text{supp}(\mu) \neq \text{supp}(\mu)^{max}$, $V(\mu) < b^f(\mu)$.

*Proof.* Let $E = \text{supp}(\mu)$. When $E \neq E^{max}$, for any $\omega \in E^{max}$, by Lemma 10, $V(\mu) < V(\delta(\omega)) = \max_{\omega' \in E} V(\delta(\omega')) = b^f(\mu)$. $\square$

Because of Proposition 3 and Lemma 5, for each PBE $(\sigma, \rho, \phi)$, and each $\omega \in \Omega$, $m \in \text{supp}(\sigma(\omega))$, it must be true that $\text{supp}(\phi(m)) = \text{supp}(\phi(m))^{max}$, and so $\omega \in \text{supp}(\phi(m))^{max}$. Then, by Lemma 6, $r(\phi(m)) = r(\delta(\omega))$. So R is taking the optimal action under the guidance of the S's information — full disclosure is achieved.

# D Section 7

## D.1 Proof of Lemma 7

Prove by contradiction. Suppose in a PBE $\mathcal{E} = (\sigma, \rho, \phi)$, an equilibrium message $m$ induces $\phi(m) = \mu$ and $U^S(\rho(m)) = \lambda(\mu)$, but $U^S(\rho(m)) < b^o(\phi(m))$. That means there is $\omega_i \in \text{supp}(\phi(m))$ such that $\sigma(\omega_i)(m) > 0$ and $U^S(\rho(m)) < g^o(\omega_i)$. Since $g^o(\omega_i) = \pi(E_i)$, $U^S(\rho(m)) < \pi(E_i)$. However, by the definition of $\pi(E_i)$, $U^S(\rho(m_{E_i})) \geq \pi(E_i) > U^S(\rho(m))$. So the sender will deviate to $m_{E_i}$ from $m$ in $\omega_i$.

## D.2 Proof of Corollary 3

(Necessity)

Condition 1 is implied by Lemma 7. Condition 2 is proved by Theorem 1. Condition 3 is proved by contradiction. If in an inferior state S receives more than a superior state, S should pretend to be the inferior state.

(Sufficiency)

Conditions 1 and 2 are equivalent to their corresponding conditions in Theorem 1. What remains is to show that condition 3 is sufficient for the third condition in Theorem 1.

Under linear disclosure, for each $\omega \in P_l$, $\{\omega\}^* \subseteq \bigcup_{k \geq l} P_k$. For each $\mu \in \text{supp}(\tau)$ s.t. $\text{supp}(\mu) \subseteq P_l$, $\text{supp}(\mu)^* \subseteq \bigcup_{k \geq l} P_k$. If there is $\omega \in P_k \neq P_l$, it means that $k > l$. Condition 3 in Corollary 3 implies that $\lambda_k > \lambda_l$. So condition 3 in Theorem 1 holds.

## D.3 Proof of Proposition 1

The proof is divided into two steps. First, I will show that the series of conditional information policies $\{\tilde{\tau}_t\}_{t=1}^T$ exist. Second, I will show that the outcome satisfies the conditions in Corollary 3.

Let me introduce a theorem in Lipnowski and Ravid (2020) that characterizes "cheap talk" equilibrium, which I will use to find the equilibrium outcome in each partition element.

**Definition 12.** *Any value $v^* \in \mathbb{R}$ is* securable *if there exists $\tau \in \mathcal{I}$, such that for each $\mu \in \text{supp}(\tau)$, $\max V(\mu) \geq v^*$.*

**Proposition 4.** *(LR, 2019) For any value $v^* \in \mathbb{R}$ s.t. $v^* \geq \min V(\mu_0)$, $v^*$ is securable if and only if there is a pair $(\tau, \lambda)$ such that for any $\mu \in \text{supp}(\tau)$, $\lambda(\mu) \in V(\mu)$ and $\lambda(\mu) = v^*$.*

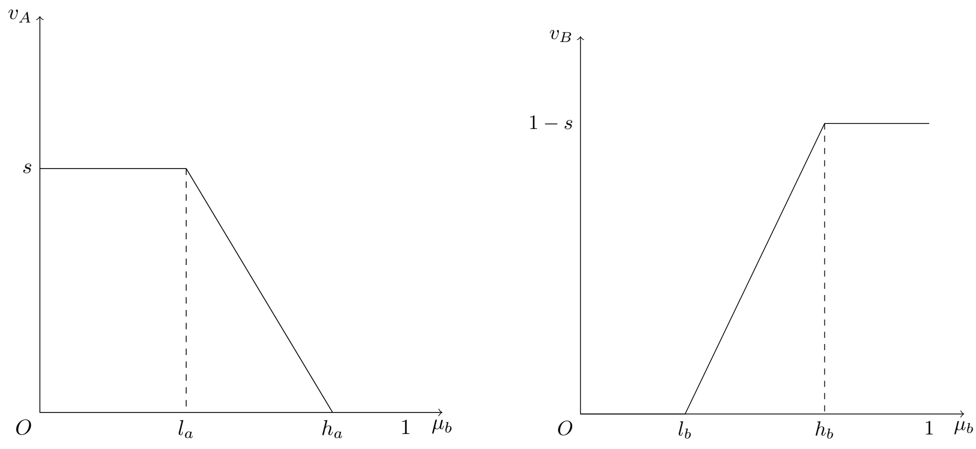
In each step $k$, if it is in case $k-2$ or case $k-3$, there is no existence problem. So I only need to discuss case $k-1$, where $v(\mu_0(\cdot|P_1^{(k-1)})) < c_2^{(k-1)} = c_1^{(k)}$. The discussion further breaks down to two situations depending on what happens in the last step $(k-1)$.

Suppose in step $(k-1)$, it is case $(k-1)-1$ or case $(k-1)-2$. In these cases $P_1^{(k-1)}$ is separated from higher states and $P_1^{(k)}$ coincides with an initial partition element $P_i^{(1)}$, for some $i$; also $c_1^{(k)} = c_i^{(1)}$. Then I show that $c_1^{(k)}$ is securable in $P_1^{(k)}$. Let $\tau_c$ be a conditional information policy in $P_1^{(k)}$ such that $\text{supp}(\tau) = \{\delta(\omega)|\omega \in P_1^{(k)}\}$ and $\tau(\delta(\omega)) = \mu_0(\omega|P_1^{(k)})$, for $\omega \in P_1^{(k)}$. By the definition of $P_i^{(1)}$, $\max V(\delta(\omega)) \geq c_i^{(1)} \geq c_1^{(k)}$, for each $\omega \in P_1^{(k)}$. So $c_1^{(k)}$ is securable in $P_1^{(k)}$, and by Proposition 4, there exists $\tilde{\tau}_t$ as described in the algorithm.

Suppose in step $(k-1)$, it is case $(k-1)-3$. In this case $P_1^{(k)} = P_1^{(k-1)} \cup P_2^{(k-1)}$. Again I can prove that $c_1^{(k)}$ is securable. Let $\tau_c$ be a conditional information policy in $P_1^{(k)}$ such that $\text{supp}(\tau_c) = \{\mu_0(\cdot|P_1^{(k-1)})\} \bigcup \{\delta(\omega)|\omega \in P_2^{(k-1)}\}$, and $\tau(\mu_0(\cdot|P_1^{(k-1)})) = \mu_0(P_1^{(k-1)}|P_1^{(k)})$ and $\tau(\delta(\omega)) = \mu_0(\omega|P_1^{(k)})$, for $\omega \in P_2^{(k-1)}$. Because it is case $(k-1)-3$, by definition $v(\mu_0(\cdot|P_1^{(k-1)})) \geq c_2^{(k-1)} = c_1^{(k)}$; and because $P_2^{(k-1)}$ is equal to some initial partition element $P_i^{(1)}$, $\max V(\delta(\omega)) \geq c_i^{(1)} \geq c_1^{(k)}$, for each $\omega \in P_2^{(k-1)}$. So $c_1^{(k)}$ is securable in $P_1^{(k)}$, and there exists $\tilde{\tau}_t$ as described in the algorithm.

So far I have shown that the algorithm will generate a series of conditional information policies $\{\tilde{\tau}_t\}_{t=1}^T$, in the rest of the proof I will show that they satisfy the conditions in Corollary 3.

Denote by $\{P_t\}_{t=1}^T$ the resulting partition elements associated with $\{\tilde{\tau}_t\}_{t=1}^T$, and by $\omega_t$ the maximal state in $P_t$ in which $c_t = v(\delta(\omega_t))$. Condition 2 is satisfied because through the algorithm, S either plays a pooling strategy or plays a strategy inducing a "cheap talk" equilibrium in the auxiliary game, so the associated payoffs are the same. Condition 3 is satisfied because based on the way each partition element is cut off, $\lambda_t < c_{t+1} \leq \lambda_{t+1}$, for each $t$.




(a) $v_A$ (b) $v_B$

Figure 7: Votes From Each Group ($\theta = \bar{\theta}$)

For condition 1, it suffices to show that for each $t$, and each $\mu \in \text{supp}(\tilde{\tau}_t)$, $\lambda(\mu) \geq b^o(\mu)$. It is given by

$$\lambda(\mu) \geq c_t = \min V(\delta(\omega_t)) \geq g^o(\omega_t) = \max_{\omega:\omega \prec \omega_t} g^o(\omega) \geq \max_{\omega:\omega \in P_t} g^o(\omega) = b^o(\mu)$$

The equality $g^o(\omega_t) = \max_{\omega:\omega \prec \omega_t} g^o(\omega)$ derives from that $g^o(\cdot)$ is increasing along the disclosure order. The reason is that the message set of a superior state includes that of an inferior state, so the lower bound should be adjusted upward. Therefore, all conditions of Corollary 3 are satisfied, and the pair $(\tilde{\tau}, \lambda)$ is a PBE outcome.

# E Section 8

## E.1 Proof of Proposition 2

First, I discuss the votes from groups $A$ and $B$ separately. Write down the vote share from group $A$ as a function of $\mu$, $v_A(\mu) = s \cdot Pr(\epsilon_A \geq -\mathbb{E}[f_A(\theta,p) \,|\, \mu])$. Similarly, I have the vote share from group $B$ as $v_B(\mu) = (1-s) \cdot Pr(\epsilon_B \geq -\mathbb{E}[f_B(\theta,p) \,|\, \mu])$. Figure 7 (a)(b) illustrate the shapes of $v_A$ and $v_B$ when $\theta = \bar{\theta}$. Both groups either completely support or oppose the incumbent when the belief is close to extremes; in the middle, however, the votes change linearly (because $\epsilon$ has a uniform distribution). The turning points for $v_i$ are $l_i$ and $h_i$, such that $l_i < h_i$, for $i = A, B$. The total vote share is $V = v_A + v_B$.

To obtain the cutoff level $l_a$, I am looking for the belief point where the voter within group $A$ most against the incumbent is indifferent. That is, $l_a$ is the solution to $\mathbb{E}[f_A(\theta,p)\,|\,\mu=(0,1-l_a,l_a)]-t=0$, i.e., $l_a=\frac{w-t}{1+w}$. Then, the cutoff level $h_a$ is where the voter within group $A$ who likes the incumbent the most is indifferent. I solve $\mathbb{E}[f_B(\theta,p)\,|\,\mu=(0,1-h_a,h_a)]-t=0$ for $h_a$. So that $h_a=\frac{t+w}{1+w}$. Similarly, I can solve for the cutoff levels for $v_B$: $l_b=\frac{1-t}{2}$ and $h_b=\frac{1+t}{2}$.

To find a PBE-1 where the incumbent reveals $\theta$, I can without loss evaluate the incumbent's value function when $\theta=\bar{\theta}$, so that $V$ can be written as a function of $\mu_b$. $V$ depends on the relative willingness of two groups to support their preferred policies. When $w<1$, it can be verified that $l_a<l_b$ and $h_a<h_b$, and there are two cases of $V$ depending on the relationship between $l_b$ and $h_a$, as illustrated by Figure 8(a)(b). When $w>1$, $l_b<l_a$ and $h_b<h_a$, and $V$ is illustrated by Figure 8(c)(d). No matter in which case, $V$ is linear in different segments.

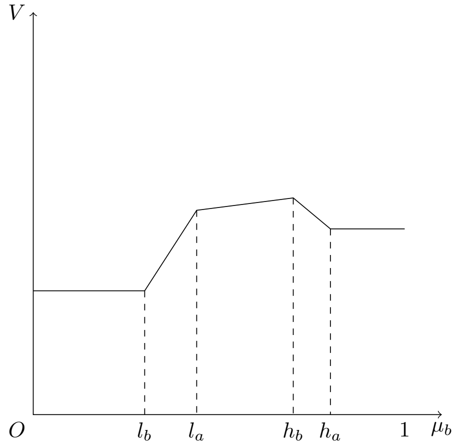
The first claim can be proved by observations that when $w>1$ (1) if $l_a\leq h_b$, then $V(h_a)=1-s$ and $V$ is strictly decreasing in $[h_b,h_a]$ (i.e., in $[\frac{1+t}{2},\frac{t+w}{1+w}]$). (2) if $l_a>h_b$, then $V(h_a)=1-s$, $V$ is strictly decreasing in $[l_a,h_a]$ and constant in $[h_b,l_a]$. Therefore, as long as $\pi\in[\frac{1+t}{2},\frac{t+w}{1+w})$, the incumbent's payoff from pooling $\bar{\theta}a$ and $\bar{\theta}b$ is larger than $(1-s)$.

The second claim can be proved by observations that when $w<1$ (1) if $l_b<h_a$, then $V$ is equal to $s$ in $[0,l_a]$, strictly decreasing in $[l_a,l_b]$, strictly increasing in $[h_a,h_b]$, and equal to $(1-s)$ in $[h_b,1]$. (2) if $l_b\geq h_a$, then $V$ is equal to $s$ in $[0,l_a]$, strictly decreasing in $[l_a,h_a]$, strictly increasing in $[l_b,h_b]$, and equal to $1-s$ in $[h_b,1]$. With these properties, it is clear that $V<1-s$, in $[0,h_b)$, and $V=1-s$, in $[h_b,1]$.

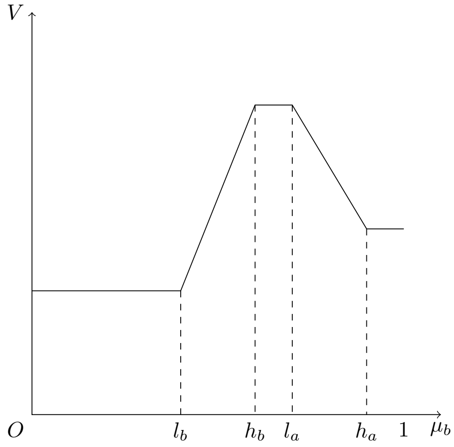
Because as $\mu_i$ increases, the value of $V$ drops, that means for each $\mu\in\Delta(\Omega)$ such that $\mu_i\in(0,1)$, $V(\mu)<V(0,1-\pi,\pi)\leq s=b(\mu)$. By Lemma 4, the belief point $\mu$ does not permit any value reaching the lower bound for equilibrium outcomes. Hence, in every equilibrium $\theta$ is revealed and it is a PRE-1. The incumbent cannot get more than $(1-s)$ in $\bar{\theta}b$ and more than 0 in $\underline{\theta}$.

The same result holds when $w=1$. In this case, $l_a=l_b$ and $h_a=h_b$ and $V$ is weakly increasing as shown in Figure 8(e).

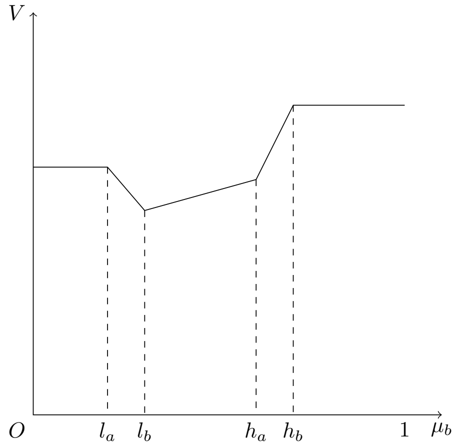
Furthermore, for any $\mu_0=(0,1-\pi,\pi)$ such that $\pi<\frac{1+t}{2}$, $V(\mu_0)<b(\mu_0)=s$, so that $\bar{\theta}a$ and $\bar{\theta}b$ cannot pool together in equilibrium. Also, I cannot find a mean-preserving spread of $\mu_0(\cdot|\theta=\bar{\theta})$ with indifferent associated payoffs, so the fully revealing equilibrium is the unique equilibrium.



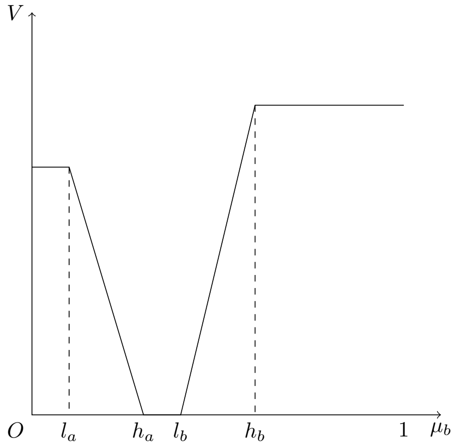

(a) $V$, $w > 1$, $l_a < h_b$



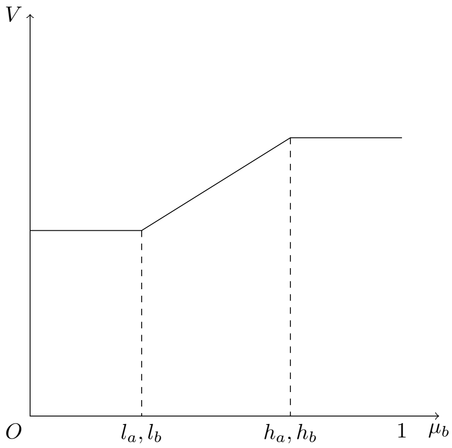

(b) $V$, $w > 1$, $l_a > h_b$




(c) $V$, $w < 1$, $l_b < h_a$




(d) $V$, $w < 1$, $l_b > h_a$




(e) $V$, $w = 1$

Figure 8: Total Vote Share ($\theta = \bar{\theta}$)